Quantum Cyber Security
Lecture 14: Post-Quantum Cryptography I

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This Lecture

1. What Post-Quantum Cryptography is?
2. Categories of Post-Quantum Secure Cryptosystems
3. Quantum Algorithms: What can a quantum adversary break
4. Quantum (Adversarial) Access To Classical Protocols
5. The Quantum Random Oracle (QRO)
6. Example: Quantum Access to Oblivious Transfer
7. Further reading: Changes in Definitions of Secure Encryption

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Definition

A classical system that withstands all quantum attacks is called **Post-Quantum Secure**

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We focus on Level 1 & Level 2

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Higher/lower confidence these are secure against QC. All less efficient/practical than used (quantumly insecure) protocols

Postquantum Standardization

Competition (4 rounds) winners (July 2022)

- **Lattices**: CRYSTALS-Kyber, CRYSTALS-Dilithium (signature), Falcon (signature)
- **Code-based**: BIKE, Classic McEliece, HQC
- **Hash-based**: SPHINCS+ (signature)
- Supersingular Elliptic Curve, Isogeny: SIKE (broken classically)

NIST standarardized (2024)

- **FIPS 203** (Federal Information Processing Standard). Encryption, based on CRYSTALS-Kyber
- **FIPS 204**. Signatures, based on CRYSTALS-Dilithium
- **FIPS 205**. Signatures, based on SPHICS+
- To-be-released **FIPS 206**. Signatures, based on FALCON (NTRU-based)

Next lectures (lattice-based earlier/simpler protocols)

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  Security parameters (key-size) for real-life implementations depend on this (quantum cryptanalysis)
- Existing quantum computers require **Quantum Error Correction** to implement most algorithms. Currently **far from breaking cryptosystems** even when there is an exponential quantum speed-up

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- Other quantum speed-ups: Simon's Algorithm, Variational Quantum Algorithms, HHL Algorithm

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- Gates are **unitary operations** (thus invertible) $U^\dagger U = \mathbb{I}$
- The final result/read-out requires also **a measurement** (non-invertible – see algorithms)

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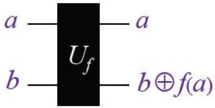

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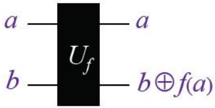

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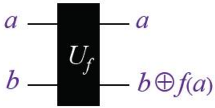

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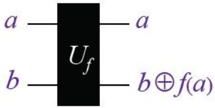

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- We can model any classical step (operation/function) as a **unitary** that takes classical inputs to classical outputs
- By linearity: superposition input gives superposition output

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  1. **Q1**: Quantum states are not communicated to honest parties

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- Assuming quantum access can be more or less realistic:
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     Examples: Encrypt superpositions in public-key setting; compute hashes of superpositions
  2. **Q2**: Honest parties receive and process quantum states

     Examples: Decrypt superpositions in public-key setting; encrypt superpositions in symmetric-key

Turning a Classical Function to Unitary

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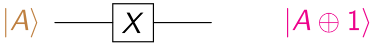
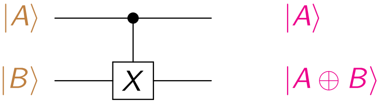
- Express the function as a Boolean circuit (AND, OR, NOT)
- Replace each gate with a reversible version of the same gate
- Replace clas gates with quantum unitaries (X, $\wedge X$, Toffoli)
- **Quantum Circuit:** on classical input returns classical output
- **Quantum Circuit:** on superpos input returns superpos output
- Behaves as Quantum Oracle (see previous lecture)

$$U_f |x\rangle |y\rangle = |x\rangle |y \oplus f(x)\rangle$$

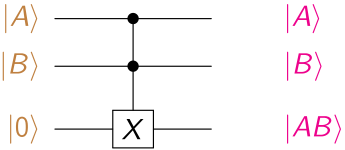
- The NOT gate:



- The reversible OR gate:



- The reversible AND gate:



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- Quantum Random Oracle (QRO): A classical random oracle that can be accessed in superposition
- Practically feasible: Given hash function, adversary can run the unitary with quantum input and obtain quantum output.

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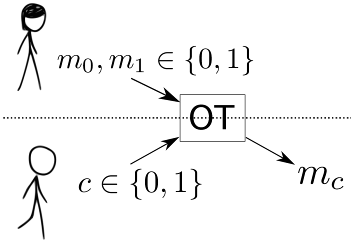
- Similar advantage for collision finding
- RO (and QRO) can be used in complicated proofs where a "programmable RO" is required.

  Further **difficulties** for QRO due to **no-cloning**!

Example of Quantum Access: 1-of-2 Oblivious Transfer



Different (classical) security definitions for OT for Bob (receiver):

1. Bob learns nothing about one message $m_{c\oplus 1}$ (guess prob 0.5)
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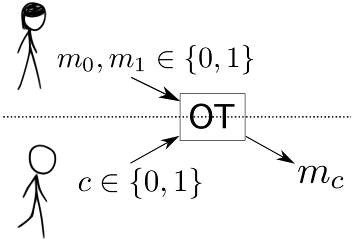

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- Classically these are equivalent
- Allowing quantum access only (2) can be achieved!

- From Bob's view the OT behaves like this gate:

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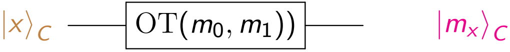

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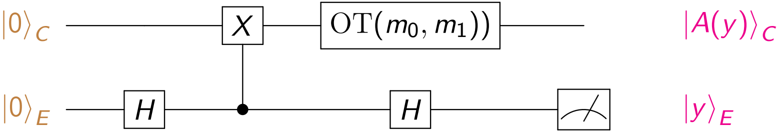
- The following circuit shows the problem:



where, $|A(y)\rangle_C := \frac{1}{\sqrt{2}}(|m_0\rangle_C + (-1)^y |m_1\rangle_C)$.

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- Definition 1 **fails**
- Definition 2 **is valid** (to guess XOR info about m_0, m_1 is lost)

Extra: Quantum Attacks to Encryptions

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- Public-Key: Essential (classical/quantum) since adversary can encrypt with public key

- Symmetric-Key: Higher Security. Quantum Access means that honest party encrypt, by default, using unitaries (preserving coherence/superpositions). **Less Realistic**

- **Chosen Ciphertext Attacks** (CCA). Gets decryption of any ciphertext he wishes (apart from challenge).

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