Quantum Cyber Security
Lecture 3: Quantum Key Distribution I

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Outline of Quantum Key Distribution Lectures

- Lecture 3: Motivation and idea of QKD; The first protocol (BB84) and intuition of security
- Lecture 8: Proper Security proof of BB84
- Lecture 9: Other QKD protocols (and quantum money)
- Lecture 11: Device-independent QKD and quantum non-locality

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Reference: Advances in Quantum Cryptography, Pirandola et al 2019, `https://arxiv.org/abs/1906.01645`

Cyber Security & Privacy: General

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Examples of tasks:

1. **Encryption:** Two parties communicate where no third party can learn anything about the content of the communication
2. **Authentication:** Parties communicate knowing that messages received come from the legitimate party (public messages)
3. **Digital Signatures:** A message with the guarantee of authenticity, integrity and non-repudiation

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Quantum Computers (when scalable) can break computationally secure cryptosystems (RSA, DSA, ECDSA)

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- Message to be sent $x = x_1 x_2 \cdots x_n$ called **plaintext**
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  3. Encryption: Bitwise addition modulo 2 of the plaintext and the secret key: $c = c_1 c_2 \cdots c_n := (x_1 \oplus k_1)(x_2 \oplus k_2) \cdots (x_n \oplus k_n)$
  4. Decryption: Bitwise addition modulo 2 of the ciphertext and the secret key: $(c_1 \oplus k_1)(c_2 \oplus k_2) \cdots (c_n \oplus k_n) =$
     $= (x_1 \oplus k_1 \oplus k_1)(x_2 \oplus k_2 \oplus k_2) \cdots (x_n \oplus k_n \oplus k_n) = x_1 x_2 \cdots x_n = x$

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     Example: $x = 1011$, $k = 0110$

     Encryption: $c = (1 \oplus 0)(0 \oplus 1)(1 \oplus 1)(1 \oplus 0) = 1101$

     Decryption: $(1 \oplus 0)(1 \oplus 1)(0 \oplus 1)(1 \oplus 0) = 1011 = x$

The Task: Key Distribution Background

Inf Theor Sec **Encryption**: Large Secret Key (One-Time-Pad)

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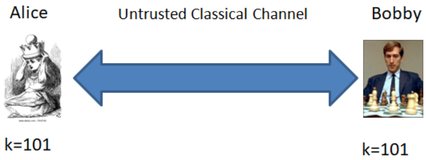

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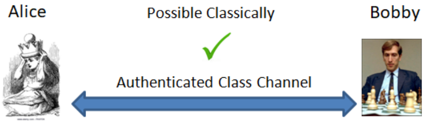

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Alice

s=110010010

Bobby

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Two spatially separated parties want to share a Large Secret Key

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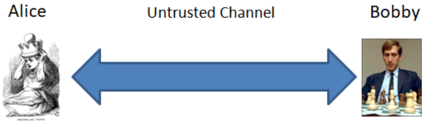

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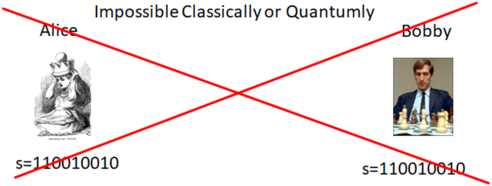

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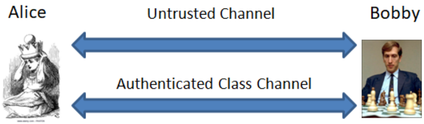

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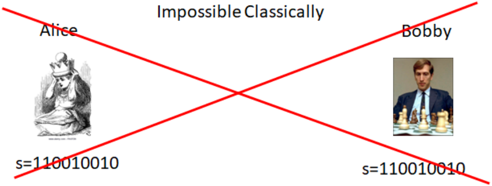

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Possible Quantumly

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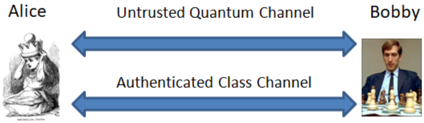
What Quantum Key Distribution Offers



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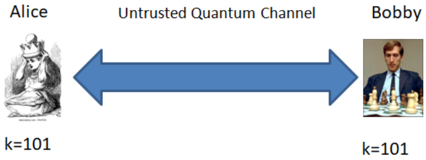

Replace Auth Class Channel with Short Key k

What Quantum Key Distribution Offers

QKD uses untrusted quantum communication and achieves:

Information Theoretic Secure **Secret Key Expansion**

What Quantum Key Distribution Offers

From **Short-Key** sufficient for Inf Theor Sec **Authentication**

Obtain **Long-Key** sufficient for Inf Theor Sec **Encryption**

Is Happening Now!

QKD is commercially available **currently**

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Satellite QKD

The BB84 Protocol

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Alice

- Sends a string of qubits each from the set $\{|h\rangle, |v\rangle, |+\rangle, |-\rangle\}$
- For each position (i) chooses randomly pair of bits $(a^{(i)}, x^{(i)})$
- $x^{(i)}$ selects the basis: $x^{(i)} = 0 \to \{|h\rangle\,, |v\rangle\}$; $x^{(i)} = 1 \to \{|+\rangle\,, |-\rangle\}$
- $a^{(i)}$ selects state: $a^{(i)} = 0 \to \{|h\rangle \text{ or } |+\rangle\}$; $a^{(i)} = 1 \to \{|v\rangle \text{ or } |-\rangle\}$
- Stores string of pairs: $(a^{(1)}, x^{(1)}), (a^{(2)}, x^{(2)}), \cdots, (a^{(n)}, x^{(n)})$

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Bob

- For each qubit (i) chooses randomly basis $y^{(i)}$ and measures
- Obtains result $b^{(i)}$: $(b^{(1)}, y^{(1)}), (b^{(2)}, y^{(2)}), \cdots, (b^{(n)}, y^{(n)})$

The BB84 Protocol

Only part that quantum was required!

The correlations between $a^{(i)}$'s and $b^{(i)}$'s and the bound on correlations these bit-strings have with **any** bit-string Eve can produce are **impossible to achieve classically** (see next)

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- Alice/Bob announce the bases $x^{(i)}, y^{(i)}$ ONLY
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- **Parameter Estimation Phase**
  They choose fraction f of the raw key **randomly** and announce $a^{(i)}, b^{(i)}$ to estimate the correlation of their strings:
  QBER – Quantum-Bit Error Rate
  Also can bound the correlation third parties have

Example:

Obtaining the Raw Key

Key value a	0	0	1	1	0
Encoding x	0	1	1	0	1
BB84 state sent by Alice	$\|h\rangle$	$\|+\rangle$	$\|-\rangle$	$\|v\rangle$	$\|+\rangle$
Measurement basis y by Bob	0	0	1	1	0
Measurement outcome b	0	1	1	1	1
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- Measurements affect the quantum state – can **detect** amount of **eavesdropping** and abort if high (more than 11% QBER)
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  Cannot intercept, copy and resend! **Ideas for attacks?**

Security: Intuition and Attempted Attack

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Question

What about intercept, measure and resend?

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- Alice and Bob detect 25% **QBER**, i.e. $p_1 \times p_2 = 1/4$

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Can bound correlations of E with A, B given estimated correlation (QBER) of A, B from Parameter Estimation

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If **QBER** low then A, B more correlated than A, E or B, E.

$$H(A:B) > H(A:E)$$

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Privacy Amplification (PA): Distil shorter key completely secret from Eve (use universal hash functions to amplify privacy)

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- If (A, B) "correlation" is higher than (A, E) then it **is possible** for Alice and Bob to distil an (identical) bit-string A'' totally secret from Eve (using IR & PA)
- The key-rate R, highest possible noise-tolerance and maximum distance possible all depend on the advantage $H(A:B) - H(A:E)$

Insights to Remember

- QKD achieves ITS **secret key expansion**
- QKD uses classical authenticated channel
- BB84 requires sending/measuring **single qubits** in **two bases**
- Eavesdropping is detected in **Parameter Estimation Phase**
- If eavesdropping is high (QBER above threshold) we **abort**
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Satellite QKD is real!

https://www.youtube.com/watch?v=YYbp-v4W_yg