Introduction to Quantum Programming and Semantics: tutorial 2

Exercise 2.1. Let M be a *commutative monoid*, which is a monoid M satisfying $mn = nm$ for all $m, n \in M$. Show that regarding M as a category, M is symmetric monoidal with the tensor product defined as $* \otimes * = *$ and $m \otimes n = mn$.

Exercise 2.2. Let $\mathbb{C}$ be a monoidal category, and let $f, g \colon I \longrightarrow I$ be morphisms in $\mathbb{C}$. Draw the following in the graphical language:

1. $g \circ f$
2. $f \circ g$
3. $f \otimes g$
4. $g \otimes f$

Exercise 2.3. Using the previous exercises 2.1 and 2.2, show that if a category with a single object is monoidal then it is symmetric monoidal.

Exercise 2.4. A *discrete category* is a category where all the morphisms are identities. Any set X can be regarded as a discrete category whose objects are the elements of X. Show that a monoid M regarded as a discrete category is a monoidal dagger category whose tensor product is defined by the multiplication of the monoid.

Exercise 2.5. Check that the terminology list on the p.15 of the slides is consistent with the usual definitions for **FHilb**.

Exercise 2.6. (Challenge) Let **Span** be the category whose objects are sets, and whose morphisms $A \longrightarrow B$ are triples (X, f, g) where X is a set, and $f \colon X \longrightarrow A$ and $g \colon X \longrightarrow B$ are functions. The composition of morphisms $(X, f_0, g_0) \colon A \longrightarrow B$ and $(Y, f_1, g_1) \colon B \longrightarrow C$ is defined by $(X \times_B Y, f_0\pi, g_1\pi')$ where $X \times_B Y \coloneqq \{(x, y) \in X \times Y \mid g_0(x) = f_1(y)\}$, and π and π' are the projections from $X \times_B Y$ to X and Y respectively.
Show that **Span** is a symmetric monoidal dagger category with the tensor product defined by the product of sets. Also show that **Span** has entangled states.