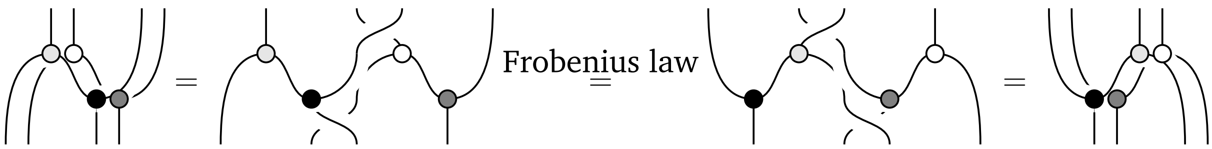
Introduction to Quantum Programming and Semantics: solutions

Exercise 5.1

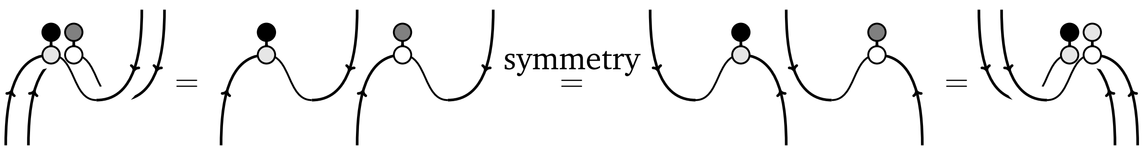
(a) Follows directly from the coherence theorem.

(b) We have seen before that the tensor product of a monoid is again a monoid. The same holds for comonoids. It is left to verify the Frobenius law:



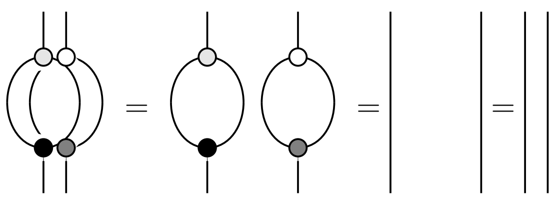
(1)

(c) We will use the fact that the tensor product $A \otimes B$ of two spaces A, B that have duals A^*, B^*, is dual to the tensor product $A^* \otimes B^*$. We use the alternative definition of symmetric Frobenius structures in symmetric monoidal categories; however, it can also be shown directly.



(2)

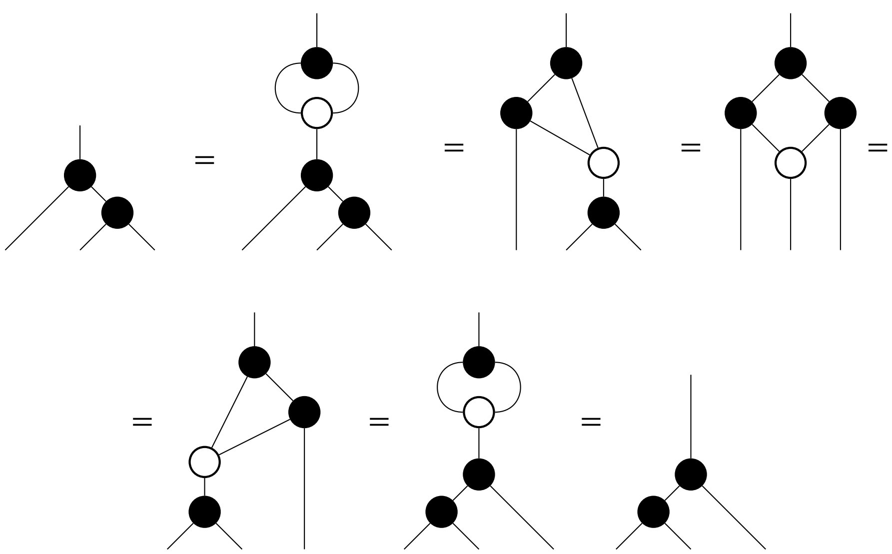
(d) The tensor product of commutative frobenius structures is again a commutative frobenius structure by (a) and an argument similar to (b). It is left to show that the tensor product of special dagger Frobenius structures is special.



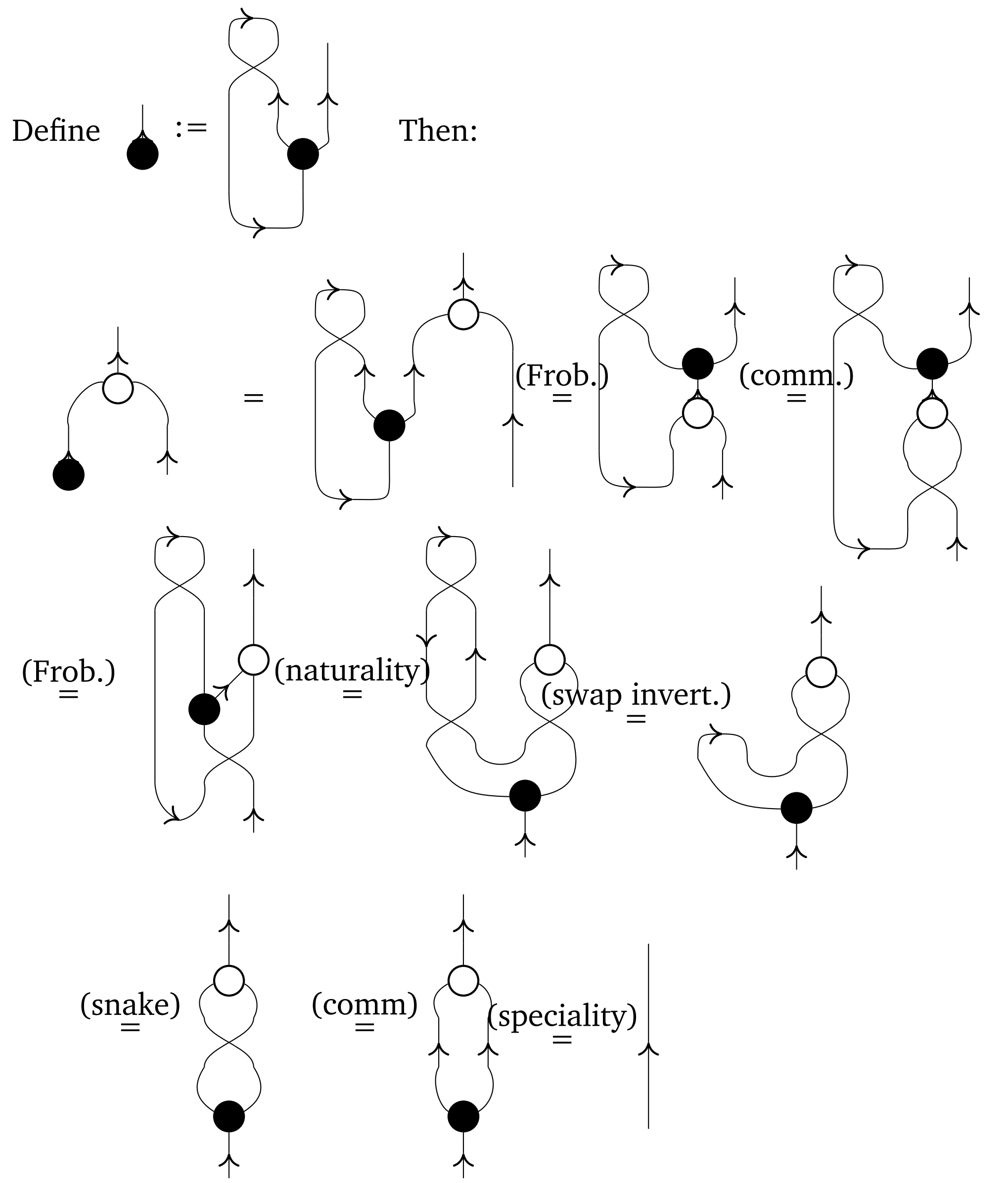
(3)

Exercise 5.2

(a)



(b)



Exercise 5.3

Note that a set containing exactly one nonzero state x_0 is trivially linearly independent. For $n > 0$, suppose $\{x_0, \ldots, x_n\}$ is a linearly dependent set of nonzero copyable states such that its subset $\{x_1, \ldots, x_n\}$ is linearly independent. Then $x_0 = \sum_{i=1}^{n} z_i x_i$ for suitable coefficients $z_i \in \mathbb{C}$. So

$$\begin{aligned} \sum_{i=1}^{n} z_i (x_i \otimes x_i) &= \sum_{i=1}^{n} z_i d(x_i) \\ &= d(x_0) \end{aligned}$$

$$= (\sum_{i=1}^{n} z_i x_i) \otimes (\sum_{j=1}^{n} z_j x_j)$$
$$= \sum_{i,j=1}^{n} z_i z_j (x_i \otimes x_j).$$

We have assumed that $\{x_1, \ldots, x_n\}$ is linearly independent; hence $z_i^2 = z_i$ for all i, and $z_i z_j = 0$ for $i \neq j$. So $z_i = 0$ or $z_i = 1$ for all i. If $z_j = 1$, then $z_i = 0$ for all $i \neq j$, so $x_0 = x_j$, contrary to assumption. So we must have $z_i = 0$ for all i. But then $x_0 = 0$, which is again contrary to assumption.

Exercise 5.6

Take the trace of: