# Reinforcement Learning

## Multi-Agent Reinforcement Learning I


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THE UNIVERSITY of EDINBURGH informatics

# Lecture Outline

Today:

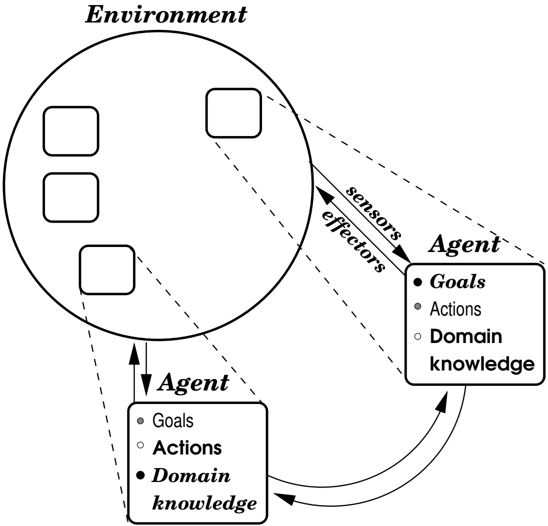
- Multi-agent systems
- Multi-agent learning and challenges
- Models of interaction
- Learning goals

Next time:

- Learning algorithms

# Multi-Agent Systems

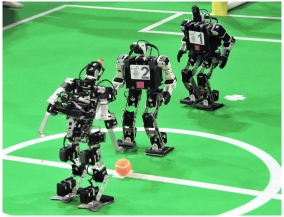
- Multiple agents interact in shared environment
- Each agent with own observations, actions, goals, ...
- Agents must coordinate actions to achieve their goals



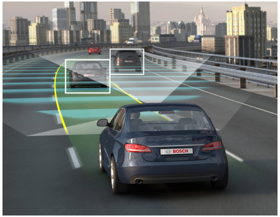
# Multi-Agent Systems – Applications

Games

Robot soccer



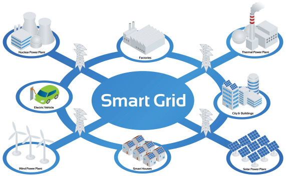
Autonomous cars



Negotiation/markets

Wireless networks

Smart grid

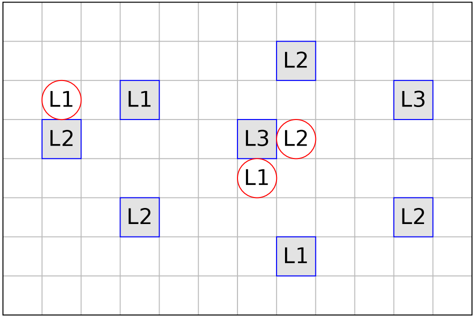


# Why Multi-Agent Systems?

**Example:** Level-based foraging

- 3 robots (circles) must collect all items in minimal time
- Robots can collect item if sum of their levels $\geq$ item level
- Action is tuple $(rob_1, rob_2, rob_3)$ with $rob_i \in \{\text{up, down, left, right, collect}\}$

  $\Rightarrow$ 125 possible actions!



# Why Multi-Agent Systems?

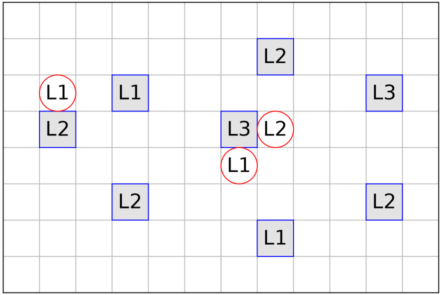
**Idea of multi-agent systems:**

*Decompose* intractable decision problem into smaller decision problems

- Use 3 agents, one for each robot

  Each agent has only 5 possible actions!

  $\Rightarrow$ *Factored action space*




**New challenge:**

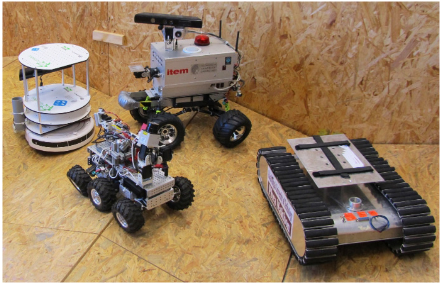
- Agents must *coordinate* actions with each other to accomplish goals

# Why Multi-Agent Systems?

More reasons for multi-agent systems:

**Decentralised control:** may not be able to control system in one central place (e.g. multiple robots working together, without communication)

**State-space reduction:** multi-agent decomposition may also reduce size of state space for individual agents (e.g. if only a subset of state features are relevant for an agent)



**Multi-agent learning:**

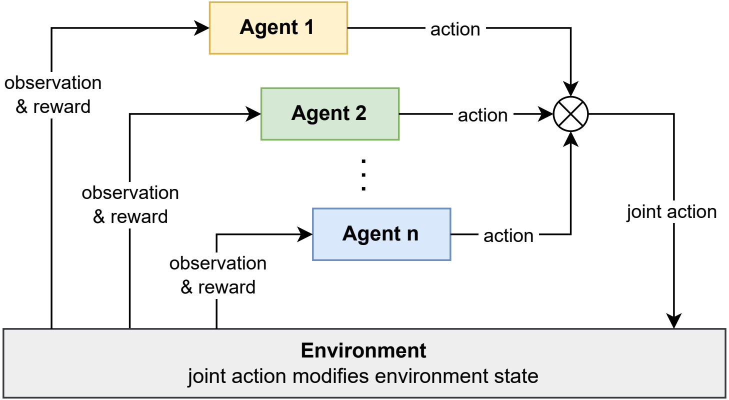
- Learning is process of improving performance via experience
- Can agents *learn* to coordinate actions with other agents?
- What to learn?

  $\Rightarrow$ How to select own actions

  $\Rightarrow$ How other agents select actions

  $\Rightarrow$ Other agents' goals, plans, beliefs, ...

# Multi-Agent Learning

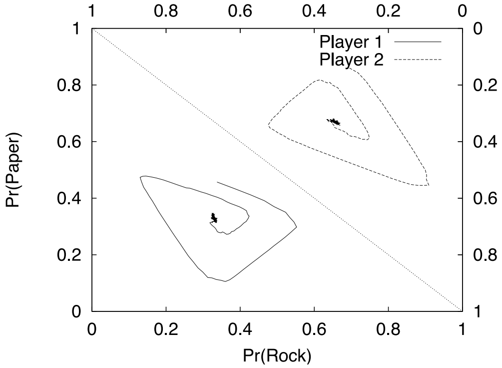


# Challenges of Multi-Agent Learning

**Non-stationary environment:**

- MDP assumes stationary environment: environment dynamics do not change over time
- If environment includes learning agents, environment becomes non-stationary from the perspective of individual agents

  $\Rightarrow$ Markov assumption broken




Moving target problem

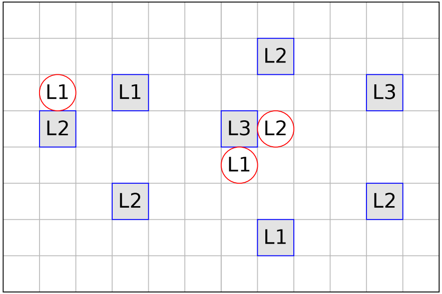
# Challenges of Multi-Agent Learning

**Multi-agent credit assignment:**

- We know (temporal) credit-assignment problem from standard RL

  $\Rightarrow$ What past actions led to current reward?
- Now we must also ask: *whose* actions led to current reward?

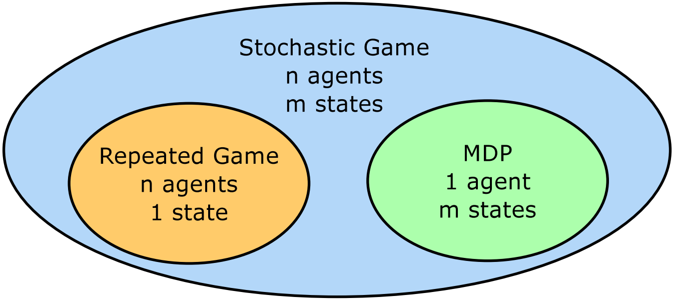
**Example:** If the two agents in centre collect L3 item, everyone gets +1 reward. How do agents know that the agent on the left did not contribute to the reward?



# Multi-Agent Models

Standard models of multi-agent interaction:

- Normal-form game
- Repeated game
- Stochastic game



# Normal-Form Game

**Normal-form game** consists of:

- Finite set of agents $N = \{1, ..., n\}$
- For each agent $i \in N$:
  - Finite set of actions $A_i$
  - Reward function $u_i : A \to \mathbb{R}$, where $A = A_1 \times ... \times A_n$ (joint action space)

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Given policy profile $(\pi_1, ..., \pi_n)$, expected reward to $i$ is

$$U_i(\pi_1, ..., \pi_n) = \sum_{a \in A} u_i(a) \prod_{j \in N} \pi_j(a_j)$$

# Normal-Form Game: Prisoner's Dilemma

**Example:** Prisoner's Dilemma

- Two prisoners are interrogated in separate rooms
- Each prisoner can **Cooperate** (C) or **Defect** (D)
- Reward matrix:

Agent 2

| | | C | D |
|---|---|---|---|
| Agent 1 | C | -1,-1 | -5,0 |
| | D | 0,-5 | -3,-3 |

# Normal-Form Game: Rock-Paper-Scissors

Example: Rock-Paper-Scissors

- Two players, three actions
- **Rock** beats **Scissors** beats **Paper** beats **Rock**
- Reward matrix:

Agent 2

| Agent 1 | | R | P | S |
|---|---|---|---|---|
| | R | 0,0 | -1,1 | 1,-1 |
| | P | 1,-1 | 0,0 | -1,1 |
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# Repeated Game

Learning is to improve performance via experience

- Normal-form game is single interaction $\Rightarrow$ *no experience!*
- Experience comes from repeated interactions

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**Repeated game:**

- Repeat the same normal-form game for time steps $t = 0, 1, 2, 3, \ldots$
- At time $t$, each agent $i$...
  - selects policy $\pi_i^t$
  - samples action $a_i^t$ with probability $\pi_i^t(a_i^t)$
  - receives reward $u_i(a^t)$ where $a^t = (a_1^t, \ldots, a_n^t)$
- Learning: modify policy $\pi_i^t$ based on history $H^t = (a^0, a^1, \ldots, a^{t-1})$

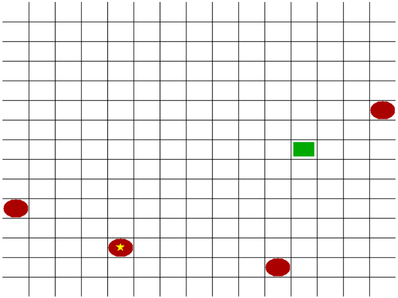
# Stochastic Game

Agents interact in shared environment

- Environment has states, and actions have effect on state
- Agents choose actions based on observed state

**Example:** Predator-prey

- Predator agents (red) must capture prey
- State: agent positions
- Actions: up, down, left, right



# Stochastic Game

**Stochastic game** (or Markov game) consists of:

- Finite set of agents $N = \{1, ..., n\}$
- Finite set of states $S$
- For each agent $i \in N$:
  - Finite set of actions $A_i$
  - Reward function $u_i : S \times A \to \mathbb{R}$, where $A = A_1 \times ... \times A_n$
- State transition probabilities $T : S \times A \times S \to [0, 1]$

*Generalises MDP to multiple agents*

# Stochastic Game

Game starts in initial state $s^0 \in S$

At time $t$, each agent $i$...

- Observes current state $s^t$
- Chooses action $a_i^t$ with probability $\pi_i(s^t, a_i^t)$
- Receives reward $u_i(s^t, a_1^t, ..., a_n^t)$

Then game transitions into next state $s^{t+1}$ with probability $T(s^t, a^t, s^{t+1})$

Repeat $T$ times or until terminal state is reached

$\Rightarrow$ Learning is now based on *state-action history* $H^t = (s^0, a^0, s^1, a^1, ...., s^t)$

# Stochastic Game — Expected Return

Given policy profile $\pi = (\pi_1, ..., \pi_n)$, what is expected return to agent $i$ in state $s$?

$$U_i(s, \pi) = \sum_{a \in A} \left( \prod_{j \in N} \pi_j(s, a_j) \right) \left[ u_i(s, a) + \gamma \sum_{s' \in S} T(s, a, s')\, U_i(s', \pi) \right]$$

- Analogous to Bellman equation
- Discount rate $0 \leq \gamma < 1$ makes return finite

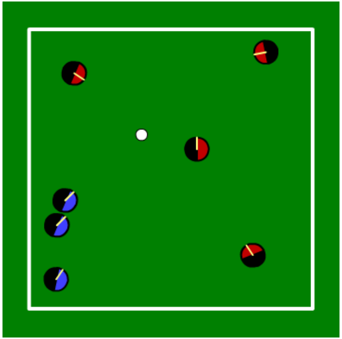
# Stochastic Game: Soccer Keepaway

**Example:** Soccer Keepaway

- "Keeper" agents must keep ball away from "Taker" agents
- State: player positions & orientations, ball position, …
- Actions: go to ball, pass ball to player, …

**Video:** Keepaway

Source: `http://www.cs.utexas.edu/~AustinVilla/sim/keepaway`



**What does it mean to solve a game?**

- If game has *common rewards*, $\forall i : u_i = u$, then solving game is like solving MDP

  $\Rightarrow$ Find policy profile $\pi = (\pi_1, ..., \pi_n)$ that maximises $U_i(s, \pi)$ for all $s$

# Solving Games

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  $\Rightarrow$ Find policy profile $\pi = (\pi_1, ..., \pi_n)$ that maximises $U_i(s, \pi)$ for all $s$

- But if agent rewards differ, $u_i \neq u_j$, what should $\pi$ optimise?

  Many solution concepts exist:

  - Minimax solution
  - Nash/correlated equilibrium
  - Pareto-optimality
  - Social welfare & fairness
  - No-regret
  - Targeted optimality & safety

# Minimax

Two-player zero-sum game: $u_i = -u_j$

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Policy profile $(\pi_i, \pi_j)$ is minimax profile if

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Reward that can be guaranteed against *worst-case* opponent

- Every two-player zero-sum normal-form game has minimax profile (von Neumann and Morgenstern, 1944)
- Every finite or infinite+discounted zero-sum stochastic game has minimax profile (Shapley, 1953)

# Nash Equilibrium

Policy profile $\pi = (\pi_1, ..., \pi_n)$ is Nash equilibrium (NE) if

$$\forall i \; \forall \pi'_i : U_i(\pi'_i, \pi_{-i}) \leq U_i(\pi)$$

No agent can improve reward by unilaterally deviating from profile
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Every finite normal-form game has at least one NE (Nash, 1950)
(also stochastic games, e.g. Fink (1964))

- Standard solution in game theory
- In two-player zero-sum game, minimax is same as NE

# Nash Equilibrium – Example

Example: Prisoner's Dilemma

- Only NE in normal-form game is (D,D)
- Normal-form NE are also NE in infinite repeated game
- Infinite repeated game has many more NE $\rightarrow$ "Folk theorem"

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| | R | P | S |
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| R | 0,0 | -1,1 | 1,-1 |
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2. Sup-optimality
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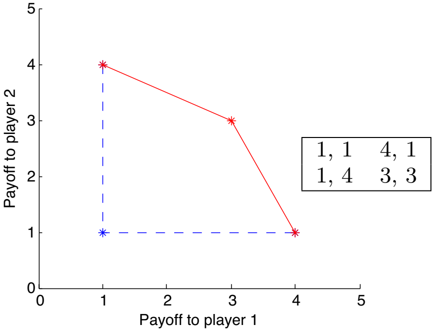
3. Incompleteness
   NE does not specify behaviours for off-equilibrium paths
4. Rationality
   NE assumes all agents are rational (= perfect reward maximisers)

# Pareto Optimum

Policy profile $\pi = (\pi_1, ..., \pi_n)$ is Pareto-optimal if there is no other profile $\pi'$ such that

$$\forall i : U_i(\pi') \geq U_i(\pi) \quad \text{and} \quad \exists_i : U_i(\pi') > U_i(\pi)$$

Can't improve one agent without making other agent worse off




Pareto-front is set of all Pareto-optimal rewards (red line)

# Social Welfare & Fairness

Pareto-optimality says nothing about social welfare and fairness

Welfare and fairness of profile $\pi = (\pi_1, ..., \pi_n)$ often defined as

$$Welfare(\pi) = \sum_i U_i(\pi) \qquad Fairness(\pi) = \prod_i U_i(\pi)$$

$\pi$ is welfare/fairness-optimal if it maximises $Welfare(\pi)/Fairness(\pi)$

$\Rightarrow$ Any welfare/fairness-optimal $\pi$ is also Pareto-optimal (Why?)

# No-Regret

Given history $H^t = (a^0, a^1, ..., a^{t-1})$, agent $i$'s regret for not having taken action $a_i$ is

$$R_i(a_i|H^t) = \sum_{\tau=0}^{t-1} u_i(a_i, a_{-i}^\tau) - u_i(a_i^\tau, a_{-i}^\tau)$$

Policy $\pi_i$ achieves no-regret if

$$\forall a_i : \lim_{t \to \infty} \frac{1}{t} R_i(a_i|H^t) \leq 0$$

(Other variants exist)

# No-Regret

Like Nash equilibrium, no-regret widely used in multi-agent learning

But, like NE, definition of regret has conceptual issues

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$\Rightarrow$ But: entire history may change if different actions taken!

- Minimising regret not generally same as maximising reward e.g. (Crandall, 2014)

## Targeted Optimality & Safety

Many algorithms designed to achieve some version of targeted optimality and safety:

- If other agent's policy $\pi_j$ is in a defined class, agent $i$'s learning should converge to best-response

$$U_i(\pi_i, \pi_j) \approx \max_{\pi_i'} U_i(\pi_i', \pi_j)$$

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- If $\pi_j$ not in class, $\pi_i$ should at least achieve safety (maximin) reward

$$U_i(\pi_i, \pi_j) \approx \max_{\pi_i'} \min_{\pi_j'} U_i(\pi_i', \pi_j')$$

**Policy classes:** non-learning, memory-bounded, finite automata, ...

# Reading (Optional)

- G. Laurent, L. Matignon, N. Le Fort-Piat. The World of Independent Learners is not Markovian. International Journal of Knowledge-Based and Intelligent Engineering Systems, 15(1):55–64, 2011
- Our RL reading list contains many survey articles on multi-agent learning: `https://eu01.alma.exlibrisgroup.com/leganto/public/44UOE_INST/lists/22066371180002466?auth=SAML&section=22066371280002466`
- AIJ Special Issue *"Foundations of Multi-Agent Learning"* (2007) `https://www.sciencedirect.com/journal/artificial-intelligence/vol/171/issue/7`

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