Reinforcement Learning

Planning and Learning

Stefano V. Albrecht, Michael Herrmann

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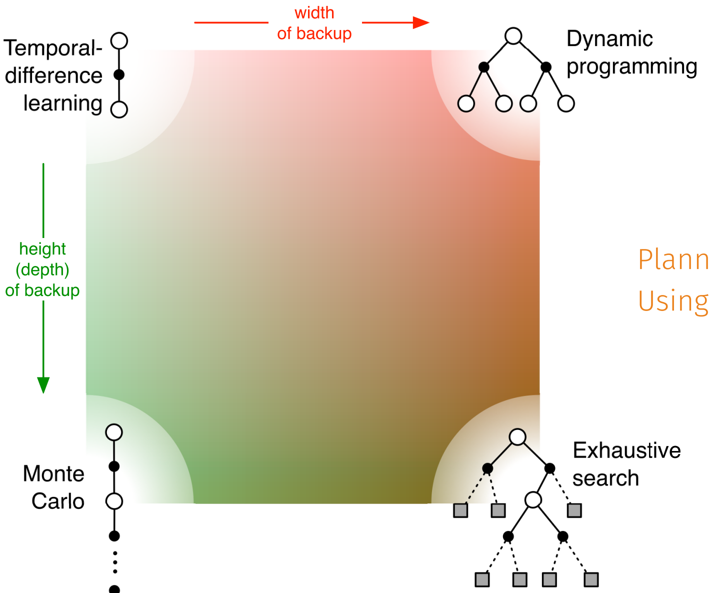
THE UNIVERSITY of EDINBURGH
informatics

Lecture Outline

- Planning in reinforcement learning
- Dyna-Q
- Rollout planning
- Monte Carlo tree search
- Offline vs online planning

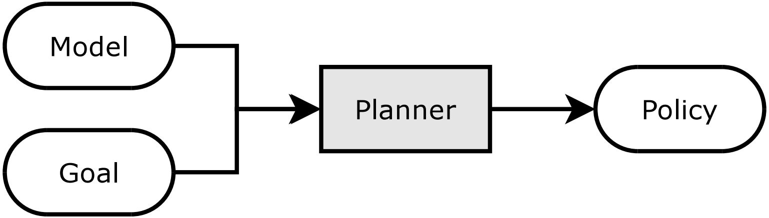
Unified View



Planning:
Using a model

Planning

Planning: any process which uses a model of the environment to compute a plan of action (policy) to achieve a specified goal



- Dynamic programming is planning: uses model $p(s', r|s, a)$

Model

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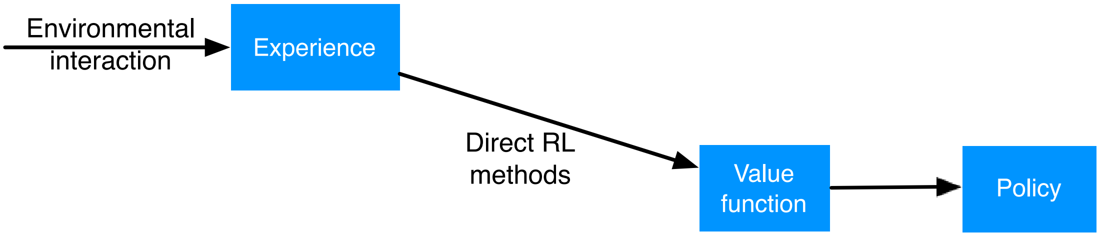
$$p(s', r|s, a) \quad \text{for all} \quad s, a, s', r$$

- Simulation (sample) model: produces sample outcomes

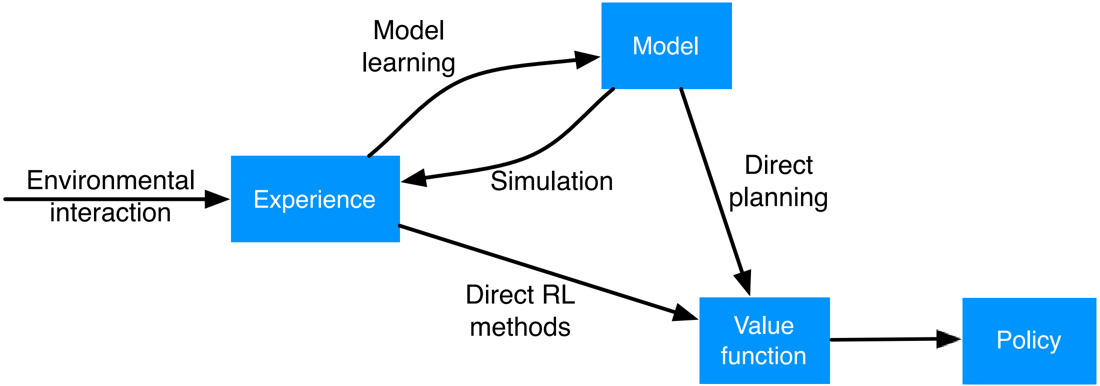
$$(s', r) \sim \hat{p}(s, a) \quad \text{s.t.} \quad \Pr\{\hat{p}(s, a) = (s', r)\} = p(s', r|s, a)$$

Simulation model usually easier to specify than distribution model

Model-free RL



Model-based RL



Dyna-Q: Integrating Planning, Learning, Acting

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$

Do forever:

(a) $S \leftarrow$ current (nonterminal) state

(b) $A \leftarrow \varepsilon$-greedy(S,Q)

(c) Execute action A; observe resultant reward, R, and state, S'

(d) $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$ ← **direct RL**

(e) $Model(S,A) \leftarrow R, S'$ (assuming deterministic environment) ← **model learning**

(f) Repeat n times: ← **planning**

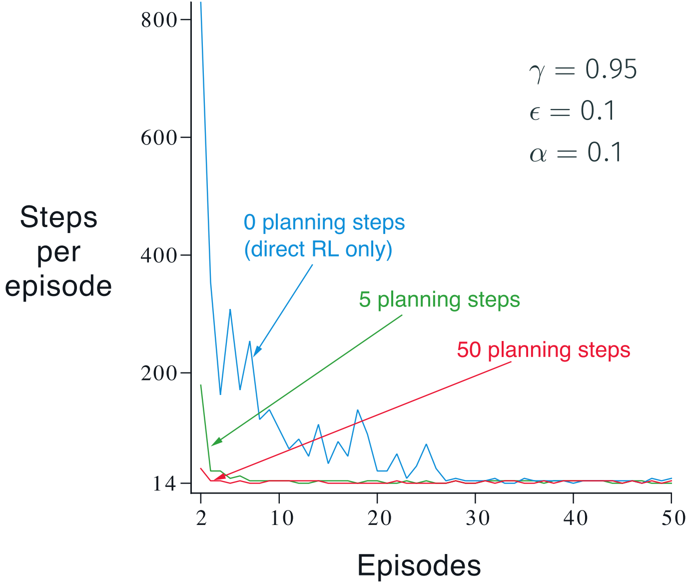
$S \leftarrow$ random previously observed state

$A \leftarrow$ random action previously taken in S

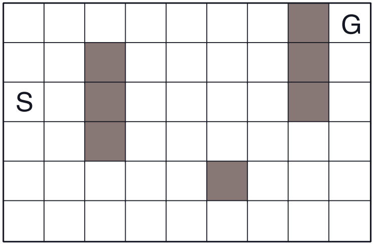
$R, S' \leftarrow Model(S,A)$

$Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$

Dyna-Q in Maze Example

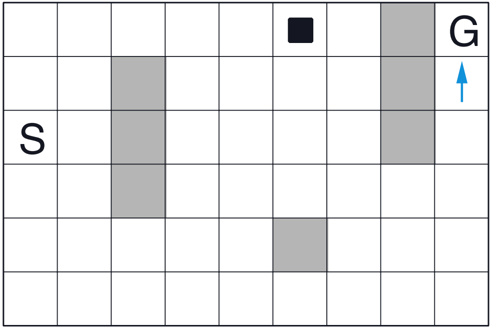


$$\gamma = 0.95$$
$$\epsilon = 0.1$$
$$\alpha = 0.1$$

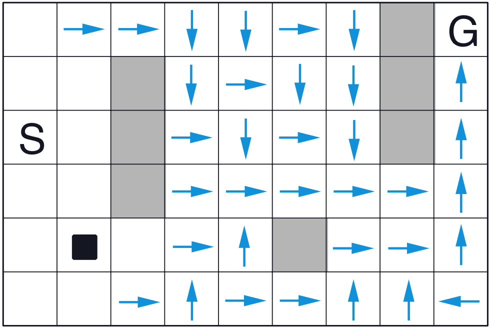


Greedy policy halfway through second episode:

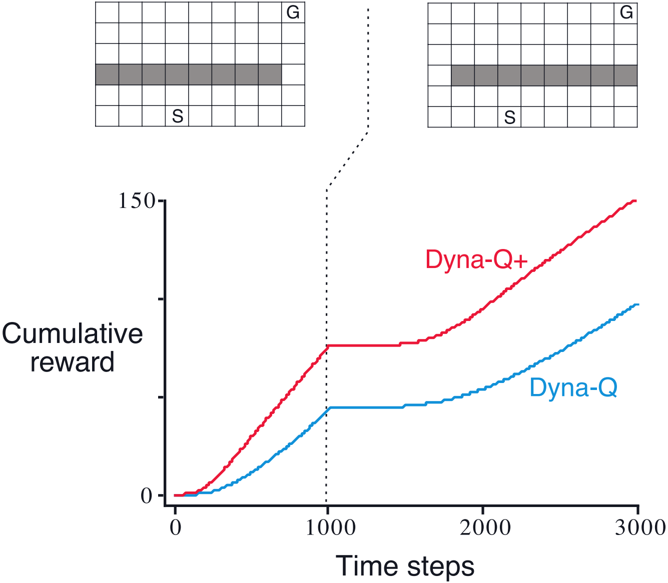
WITHOUT PLANNING (n=0)



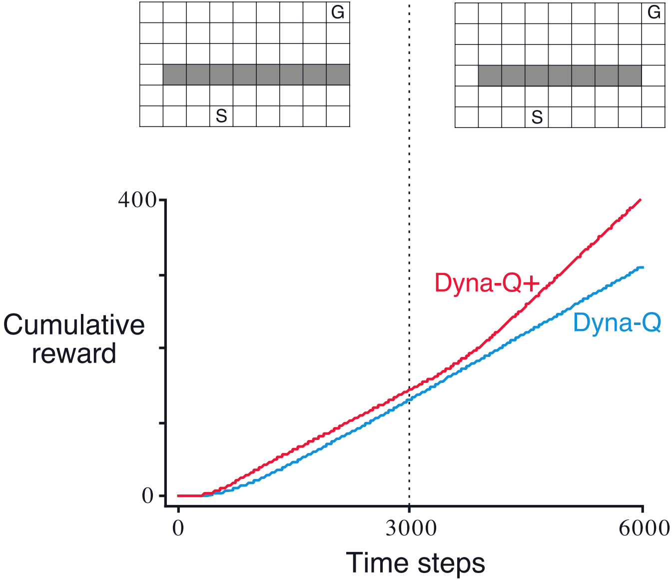
WITH PLANNING (n=50)



When the Model is Wrong: Blocking Maze



When the Model is Wrong: Shortcut Maze



Dyna-Q+

See Tutorial 6

Dyna-Q+ uses an exploration bonus heuristic:

- Keeps track of time since each state-action pair was tried in real environment
- Bonus reward is added for transitions caused by state-action pairs related to how long ago they were tried:

$$R + \kappa\sqrt{\tau}$$

time since last visiting the state-action pair

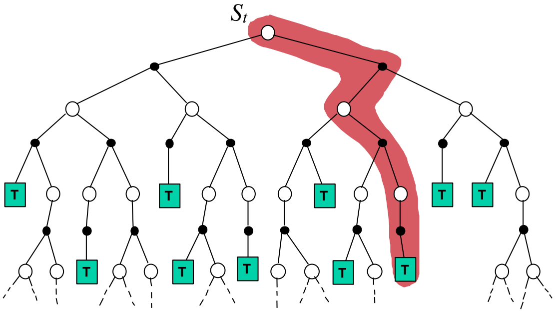
- Incentive to re-visit "old" state-action pairs

Rollout Planning

Dyna-Q uses model to reuse *past* experiences

Rollout planning:

- Use model to simulate ("rollout") *future* trajectories
- Each trajectory starts at current state S_t
- Find best action A_t for state S_t



Rollout Planning with Forward Updating

Rollout Q-planning with forward updating:

Given: simulation model *Model*
Initialise: $Q(s, a)$ for all s, a
for $t = 0, 1, 2, 3, \ldots$ **do**
    $S_t \leftarrow$ current state
    for n rollouts **do**
        $S \leftarrow S_t$
        while S is non-terminal (or fixed-length rollouts) **do**
            select action A based on $Q(S, \cdot)$ with some exploration   // *e.g.* ϵ*-greedy*
            $(R, S') \sim Model(S, A)$
            Q-update: $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$
            $S \leftarrow S'$
    select action A_t greedily from $Q(S_t, \cdot)$

Rollout Planning Optimality

If model is **correct** and under Q-learning conditions (all (s, a) infinitely visited and standard α-reduction), rollout planning learns *optimal policy*

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- Can range from slightly sub-optimal to failing to solve real task (examples?)

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Next: can we use rewards from rollouts more effectively?

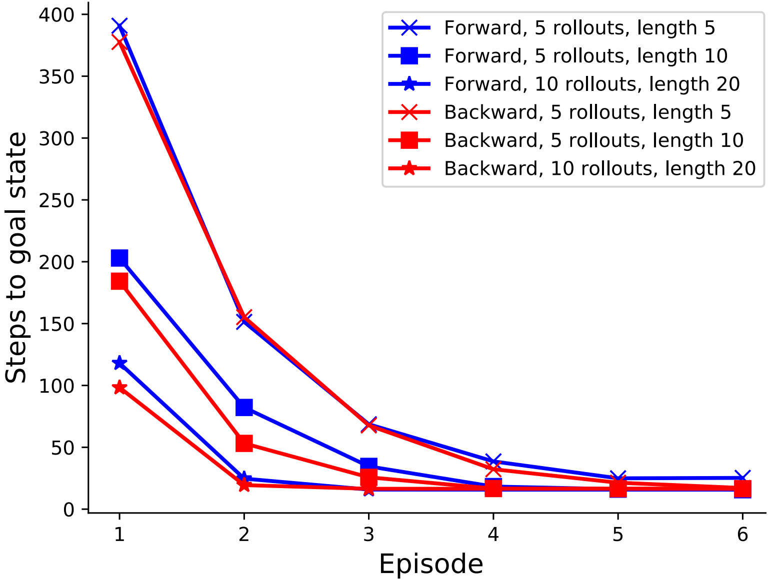
$\Rightarrow$ Back-propagate rewards

Rollout Planning with Backward Updating (Back-Propagation)

Rollout Q-planning with backward updating:

```
Given: simulation model Model
Initialise: Q(s, a) for all s, a;  LIFO stack Trace = {}
for t = 0, 1, 2, 3, ... do
    S_t ← current state
    for n rollouts do
        S ← S_t
        while S is non-terminal (or fixed-length rollouts) do      // Rollout
            select action A based on Q(S, ·) with some exploration
            (R, S') ~ Model(S, A)
            push (S, A, R, S') to Trace
            S ← S'
        while Trace not empty do                                   // Backprop
            pop (S, A, R, S') from Trace
            Q(S, A) ← Q(S, A) + α[R + γ max_a Q(S', a) − Q(S, A)]
    select action A_t greedily from Q(S_t, ·)
```

Rollout Planners in Maze Example



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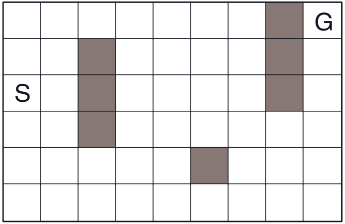
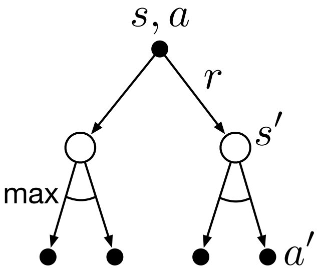


Monte Carlo Tree Search (MCTS):

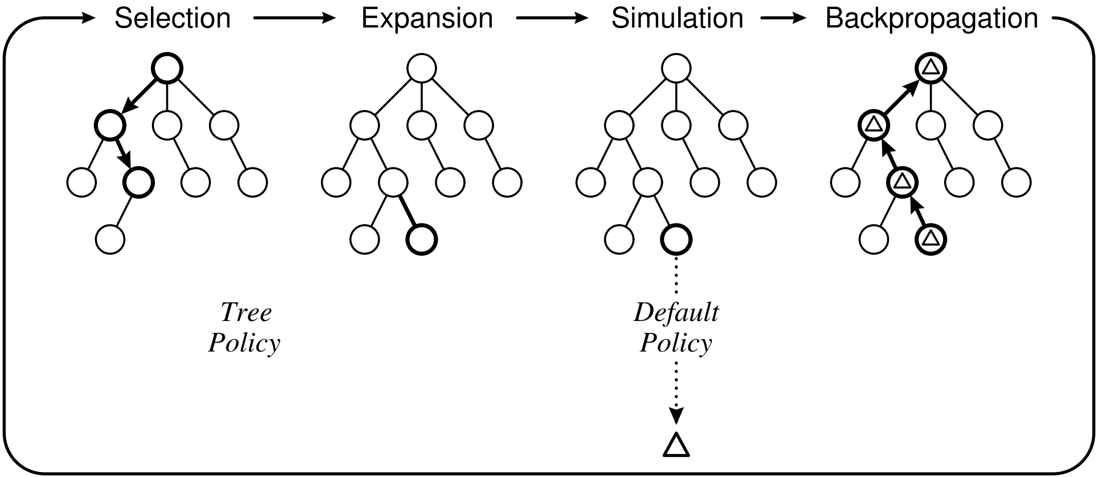
- General, efficient rollout planning with backward updating
- Stores partial Q as tree and asymmetrically expands tree based on most promising actions

Q is recursive tree structure:

$$Q(s,a) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') \mid S_t = a, A_t = a]$$



Phases of Monte Carlo Tree Search



Browne et al. (2012)

General MCTS Method

MCTS-Search(S_t):

Find node v_0 with $state(v_0) = S_t$ (or create new node)
while within computational budget **do**
  $v_l \leftarrow TreePolicy(v_0)$ *// Select node in tree and expand*
  $\Delta \leftarrow DefaultPolicy(state(v_l))$ *// Simulation steps*
  $Backprop(v_l, \Delta)$
return $action(BestChild(v_0))$ *// e.g. highest expected return; most visited child*

- Tree policy can be any exploration policy
- Backprop works just as before

Upper Confidence Bounds for Trees (UCT):

- Popular MCTS variant — easy to use and often effective
- Uses UCB action selection as tree policy, and $\alpha = 1/N(S, A)$

UCB recap: estimate upper bound on action value:

$$A \leftarrow \begin{cases} a, & \text{if } a \text{ never tried in } S \\ \arg\max_a Q(S, a) + c\sqrt{\log N(S)/N(S, a)} \end{cases}$$

- $N(S)$ is number of times state S has been visited
- $N(S, a)$ is number of times action a was selected in S

Simulation Step

Simulation step gives estimate of return at state, e.g.:

Random-DefaultPolicy(S):

```
G ← 0
while S is non-terminal do
    A ← random action (uniformly)
    (R, S′) ~ Model(S, A)
    G ← R + γG
    S ← S′
return G
```

Possible improvements:

- Average over multiple simulations
- Use domain-specific heuristic to
  – select better actions than random
  – evaluate state directly (e.g. in Chess we know that some states are better than others)

Offline Planning

Imagine you are given an MDP for a chess game against a specific opponent

Offline planning:

- Use MDP to find best policy before the actual chess game takes place (offline)
- Use as much time as needed to find policy
- Policy is complete: gives optimal action for *all* possible states

Dyna-Q and dynamic programming are suitable for offline planning

Online Planning

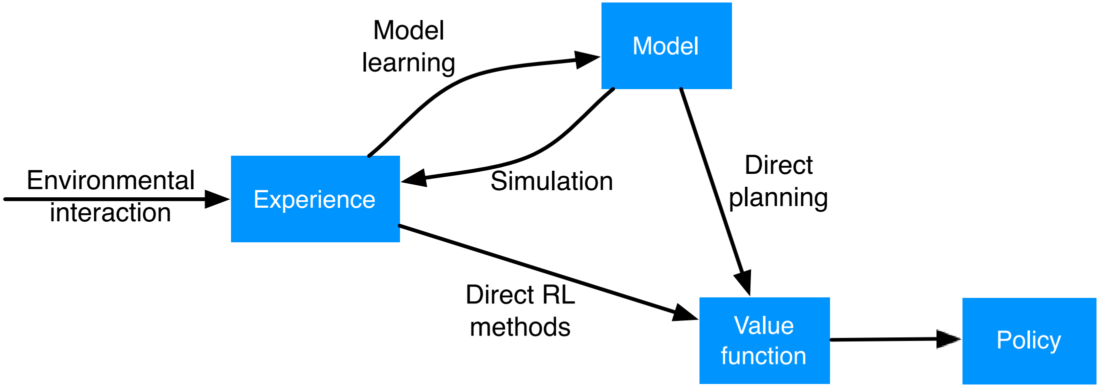
Imagine you are given an MDP for a chess game against a specific opponent

Online planning:

- Use MDP to find best policy during the actual chess game (online)
- Limited compute time budget at each state (e.g. seconds/minutes in chess)
- Policy usually incomplete: gives optimal action for *current* state

Rollout planning (including MCTS) is suitable for online planning

Paths to a Policy: Model-Based RL



Reading

Required:

- RL book, Chapter 8 (8.1–8.3, 8.10–8.11)

Optional:

- Browne et al. (2012). A Survey of Monte Carlo Tree Search Methods. IEEE Transactions on Computational Intelligence and AI in Games, Vol. 4, No. 1
- UCT paper: L. Kocsis and C. Szepesvari (2006). Bandit based Monte-Carlo Planning. European Conference on Machine Learning
- T. Vodopivec, S. Samothrakis, B. Ster (2017). On Monte Carlo Tree Search and Reinforcement Learning. Journal of Artificial Intelligence Research, Vol. 60