Reinforcement Learning

Planning and Learning

Michael Herrmann, David Abel

Based on slides by Stefano V. Albrecht

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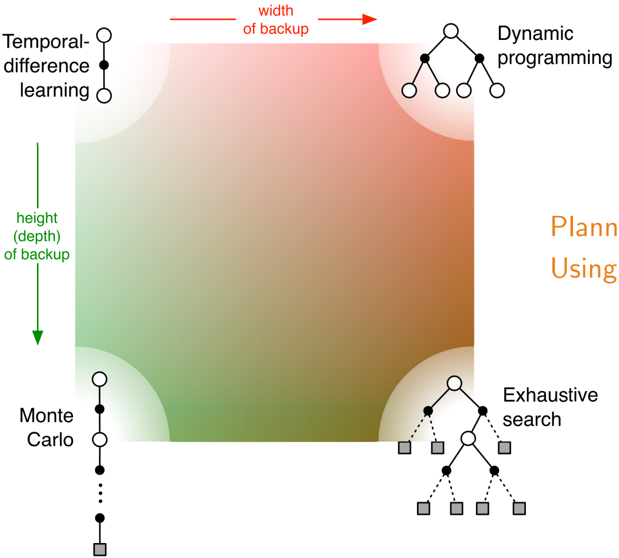
Lecture Outline

- n-step TD methods (cntd. from 4/2/25)
- Planning in reinforcement learning
- Dyna-Q
- Rollout planning
- Monte Carlo tree search
- Offline vs online planning

Unified View



Planning:

Using a model

Reminder: TD control

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\textit{terminal}) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal

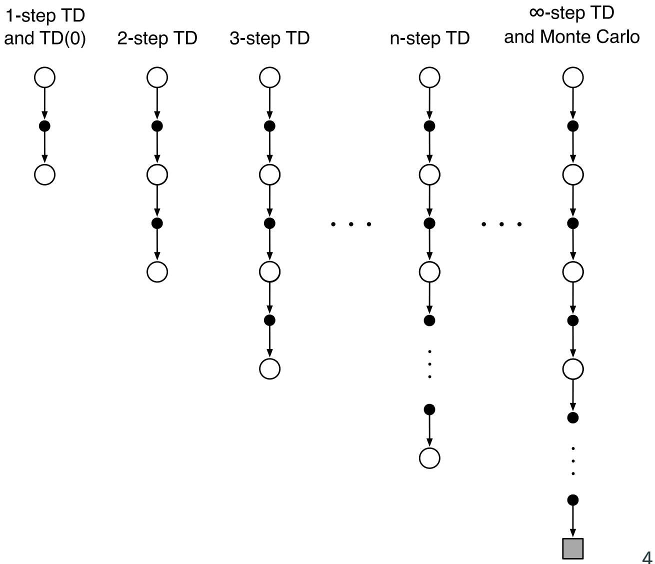
n-step TD Methods

TD(0) uses 1-step return:

$$G_{t:t+1} \doteq R_{t+1} + \gamma V_t(S_{t+1})$$

MC uses full return:

$$G_{t:\infty} \doteq \sum_{k=1}^{\infty} \gamma^{k-1} R_{t+k}$$



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MC uses full return:

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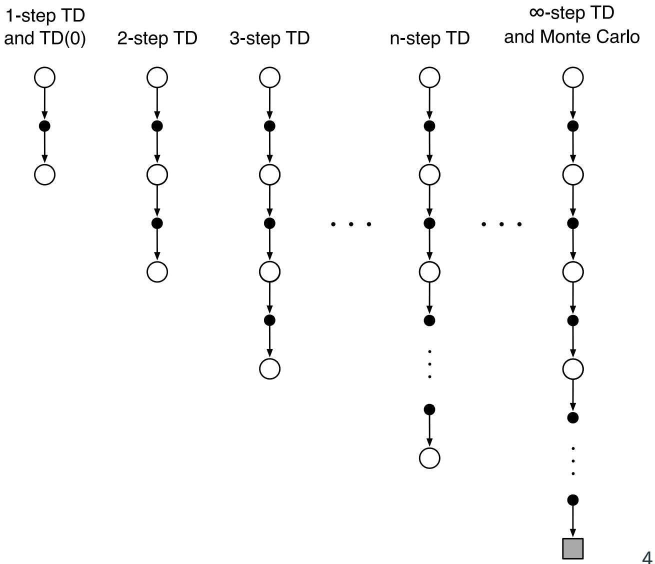
n-step return bridges TD(0) and MC:

$$G_{t:t+n} = \sum_{k=1}^{n} \gamma^{k-1} R_{t+k} + \gamma^n V_{t+n-1}(S_{t+n})$$



n-step TD Methods

n-step return:

$$G_{t:t+n} = \sum_{k=1}^{n} \gamma^{k-1} R_{t+k} + \gamma^n V_{t+n-1}(S_{t+n})$$

n-step TD uses n-step return as target:

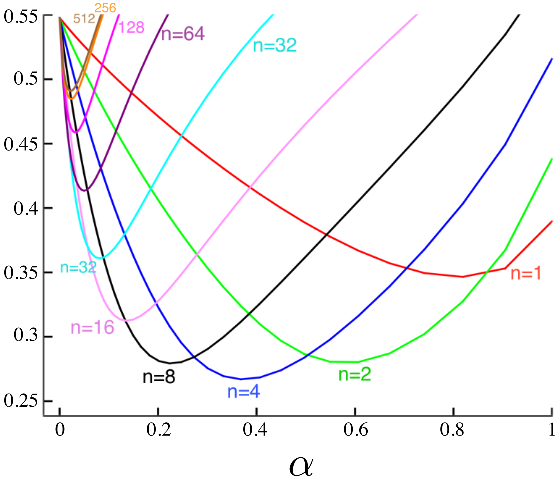
$$V_{t+n}(S_t) \doteq V_{t+n-1}(S_t) + \alpha \left[G_{t:t+n} - V_{t+n-1}(S_t) \right]$$

n-step TD Methods in Random Walk Example (see slide 14 in RL_6)

$\pi(left/right|s) = 0.5$

Average RMS error over 19 states and first 10 episodes

(Larger problem than shown in the top image)



On/Off-Policy Learning with n-Step Returns

Can similarly define *n*-step TD policy learning:

$$G_{t:t+n} = \sum_{k=1}^{n} \gamma^{k-1} R_{t+k} \; + \; \gamma^{n} Q_{t+n-1}(S_{t+n}, A_{t+n})$$

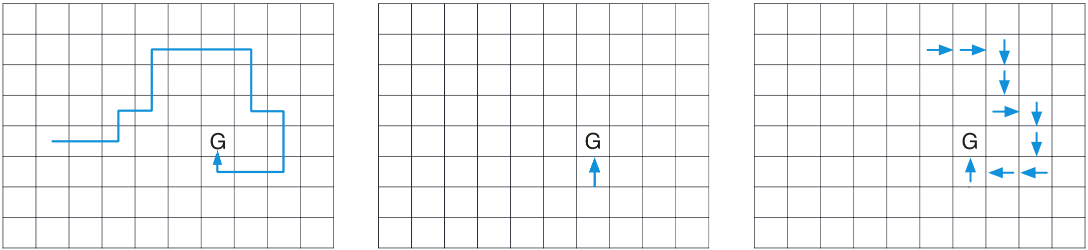
$$Q_{t+n}(S_t, A_t) \doteq Q_{t+n-1}(S_t, A_t) + \alpha \rho_{t+1:t+n} \left[G_{t:t+n} - Q_{t+n-1}(S_t, A_t) \right]$$

with importance ratio

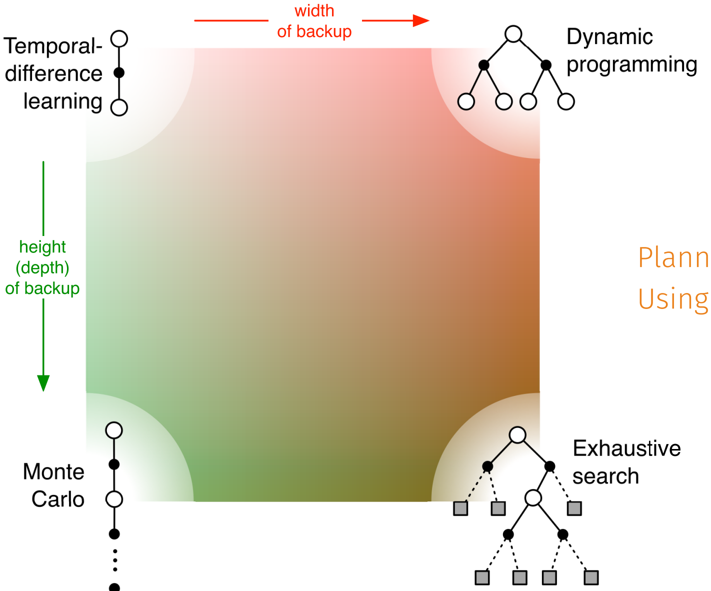
$$\rho_{t:h} \doteq \prod_{k=t}^{\min(h,T-1)} \frac{\pi(A_k|S_k)}{\mu(A_k|S_k)}$$

n-step TD Control in a Gridworld

Path taken | Action values increased by one-step Sarsa | Action values increased by 10-step Sarsa



Unified View

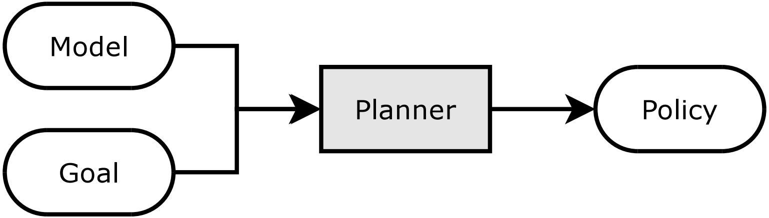


Planning:

Using a model

Planning

Planning: any process which uses a model of the environment to compute a plan of action (policy) to achieve a specified goal



- Dynamic programming is planning: uses model $p(s', r|s, a)$

Model

Model: anything the agent can use to predict how environment will respond to actions

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- Distribution model: description of all possibilities and their probabilities

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- Distribution model: description of all possibilities and their probabilities

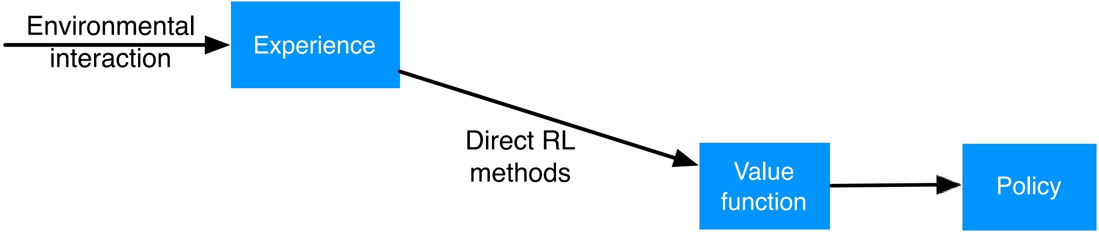
$$p(s', r | s, a) \quad \text{for all} \quad s, a, s', r$$

- Simulation (sample) model: produces sample outcomes

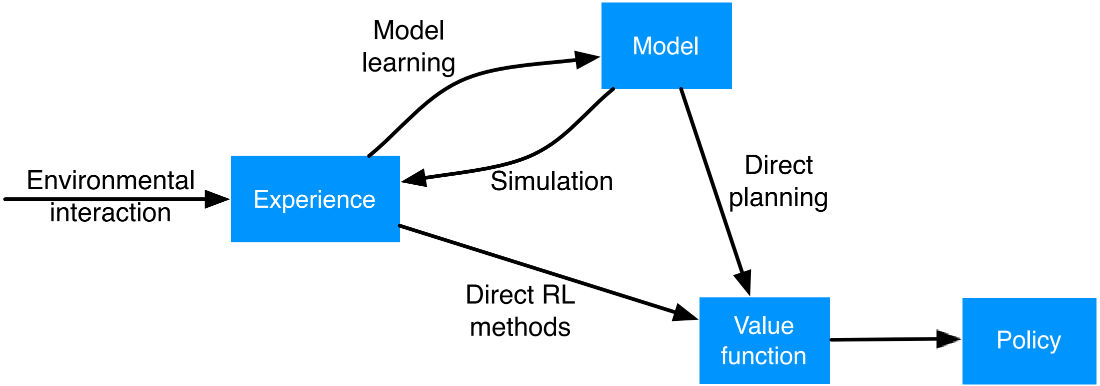
$$(s', r) \sim \hat{p}(s, a) \quad \text{s.t.} \quad \Pr\{\hat{p}(s, a) = (s', r)\} = p(s', r | s, a)$$

Simulation model usually easier to specify than distribution model

Model-free RL



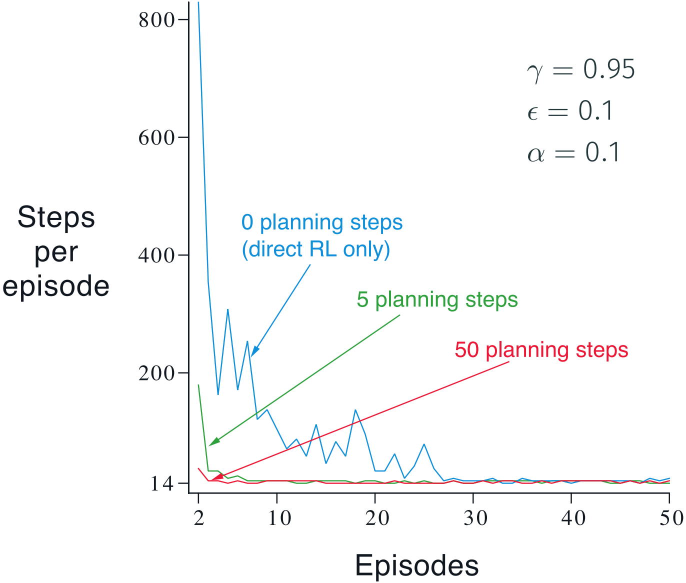
Model-based RL



Dyna-Q: Integrating Planning, Learning, Acting

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Do forever:
(a) $S \leftarrow$ current (nonterminal) state
(b) $A \leftarrow \varepsilon\text{-greedy}(S, Q)$
(c) Execute action A; observe resultant reward, R, and state, S'
(d) $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$ **direct RL**
(e) $Model(S,A) \leftarrow R, S'$ (assuming deterministic environment) **model learning**
(f) Repeat n times: **planning**
$S \leftarrow$ random previously observed state
$A \leftarrow$ random action previously taken in S
$R, S' \leftarrow Model(S,A)$
$Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$

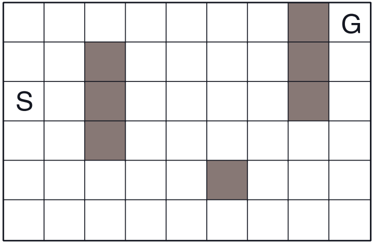
Dyna-Q in Maze Example



$\gamma = 0.95$

$\epsilon = 0.1$

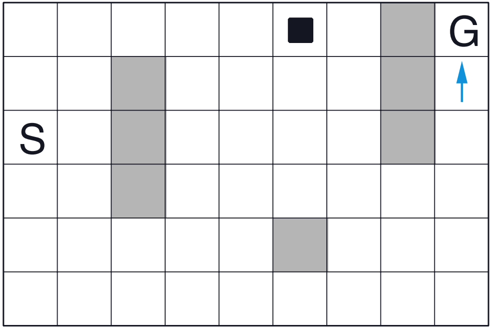
$\alpha = 0.1$



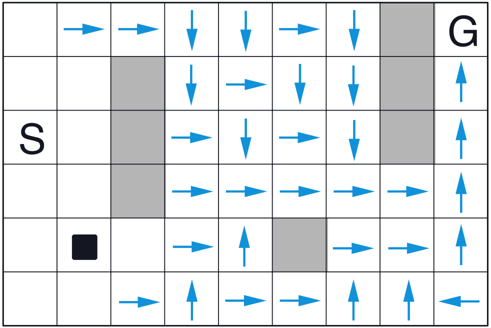
Dyna-Q in Maze Example

Greedy policy halfway through second episode:

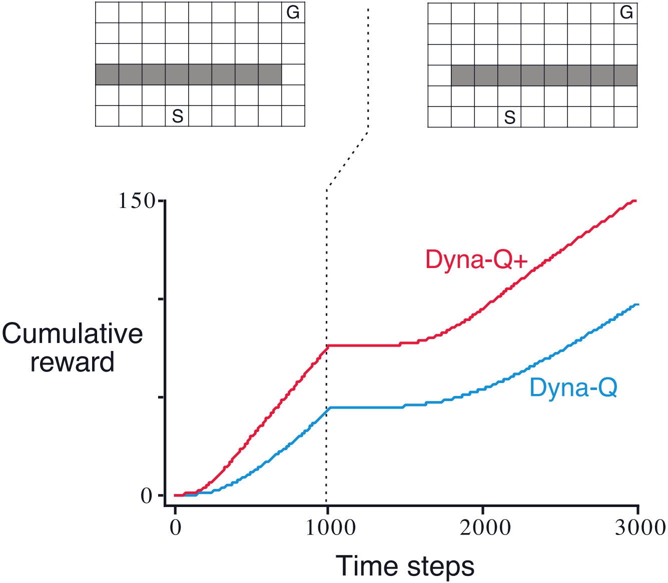
WITHOUT PLANNING (n=0)



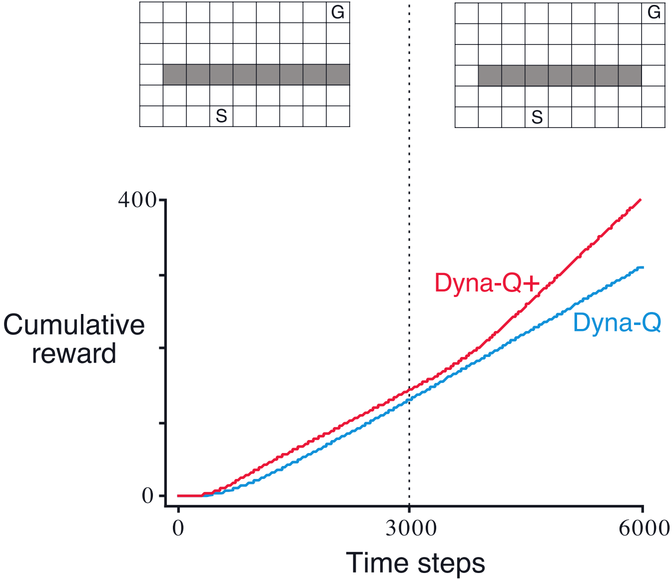
WITH PLANNING (n=50)



When the Model is Wrong: Blocking Maze



When the Model is Wrong: Shortcut Maze



Dyna-Q+

See Tutorial 6

Dyna-Q+ uses an exploration bonus heuristic:

- Keeps track of time since each state-action pair was tried in real environment
- Bonus reward is added for transitions caused by state-action pairs related to how long ago they were tried:

$$R + \kappa\sqrt{\tau}$$

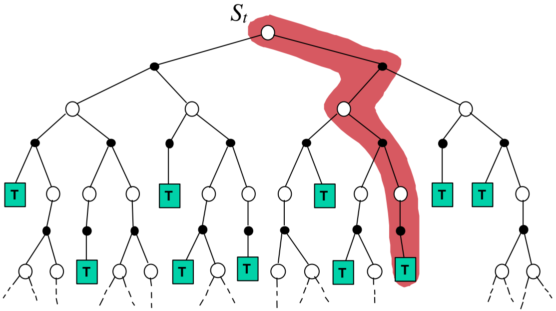
time since last visiting the state-action pair

- Incentive to re-visit "old" state-action pairs

Dyna-Q uses model to reuse *past* experiences

Rollout planning:

- Use model to simulate ("rollout") *future* trajectories
- Each trajectory starts at current state S_t
- Find best action A_t for state S_t



Rollout Planning with Forward Updating

Rollout Q-planning with forward updating:

Given: simulation model *Model*
Initialise: $Q(s, a)$ for all s, a
for $t = 0, 1, 2, 3, \ldots$ **do**
  $S_t \leftarrow$ current state
  for n rollouts **do**
    $S \leftarrow S_t$
    while S is non-terminal (or fixed-length rollouts) **do**
      select action A based on $Q(S, \cdot)$ with some exploration // *e.g. ϵ-greedy*
      $(R, S') \sim Model(S, A)$
      Q-update: $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$
      $S \leftarrow S'$
  select action A_t greedily from $Q(S_t, \cdot)$

Rollout Planning Optimality

If model is **correct** and under Q-learning conditions (all (s, a) infinitely visited and standard α-reduction), rollout planning learns *optimal policy*

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If model is **incorrect**, learned policy likely sub-optimal on real task

- Can range from slightly sub-optimal to failing to solve real task (examples?)

Rollout Planning Optimality

If model is **correct** and under Q-learning conditions (all (s, a) infinitely visited and standard α-reduction), rollout planning learns *optimal policy*

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- Can range from slightly sub-optimal to failing to solve real task (examples?)

Next: can we use rewards from rollouts more effectively?

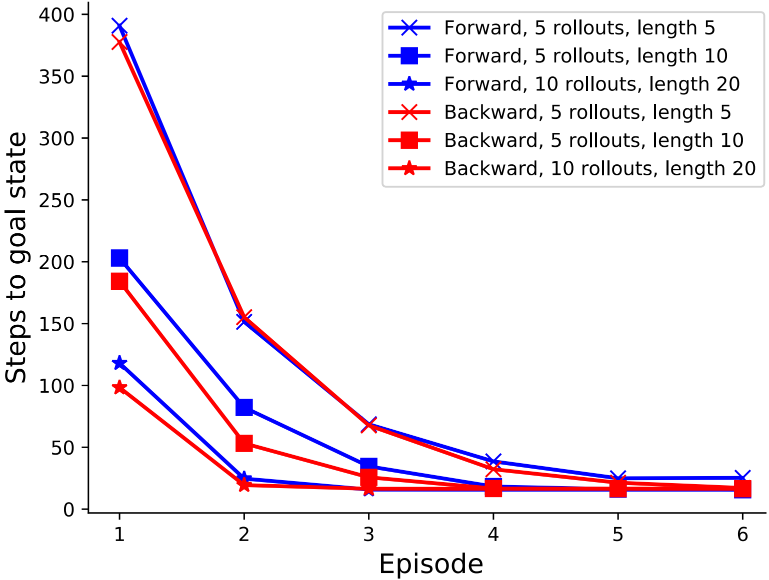
$\Rightarrow$ Back-propagate rewards

Rollout Planning with Backward Updating (Back-Propagation)

Rollout Q-planning with backward updating:

Given: simulation model *Model*
Initialise: $Q(s, a)$ for all s, a; LIFO stack $Trace = \{\}$
for $t = 0, 1, 2, 3, \ldots$ **do**
  $S_t \leftarrow$ current state
  for n rollouts **do**
    $S \leftarrow S_t$
    while S is non-terminal (or fixed-length rollouts) **do**   *// Rollout*
      select action A based on $Q(S, \cdot)$ with some exploration
      $(R, S') \sim Model(S, A)$
      push (S, A, R, S') to *Trace*
      $S \leftarrow S'$
    while *Trace* not empty **do**   *// Backprop*
      pop (S, A, R, S') from *Trace*
      $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$
  select action A_t greedily from $Q(S_t, \cdot)$

Rollout Planners in Maze Example



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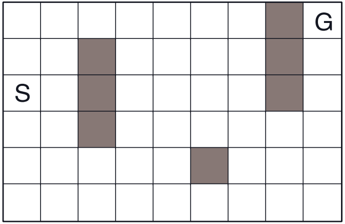
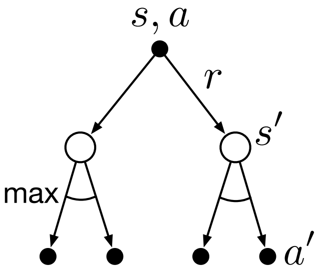


Monte Carlo Tree Search (MCTS):

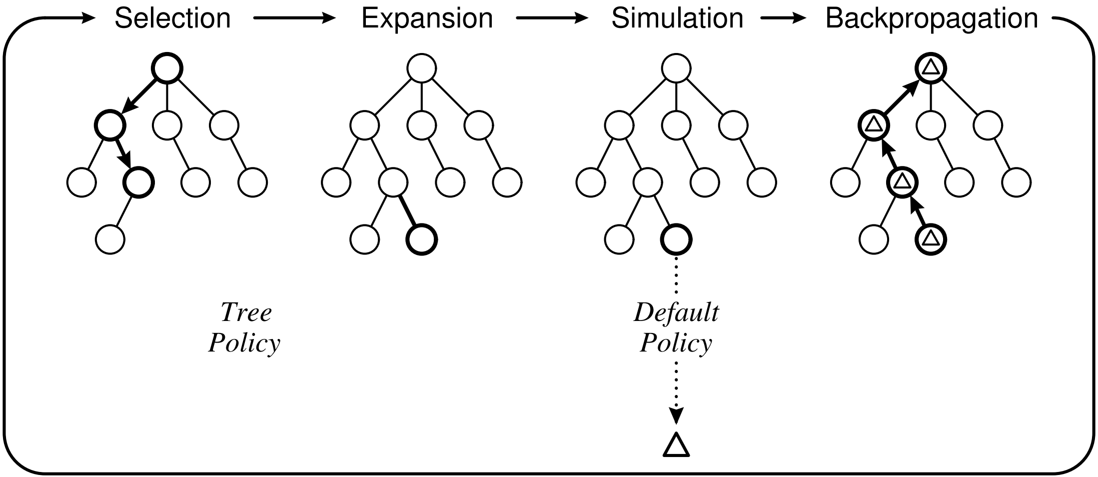
- General, efficient rollout planning with backward updating
- Stores partial Q as tree and asymmetrically expands tree based on most promising actions

Q is recursive tree structure:

$$Q(s, a) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') \mid S_t = a, A_t = a]$$



Phases of Monte Carlo Tree Search



Browne et al. (2012)

General MCTS Method

MCTS-Search(S_t):

```
Find node v_0 with state(v_0) = S_t (or create new node)
while within computational budget do
    v_l ← TreePolicy(v_0)                     // Select node in tree and expand
    Δ ← DefaultPolicy(state(v_l))             // Simulation steps
    Backprop(v_l, Δ)
return action(BestChild(v_0))   // e.g. highest expected return; most visited child
```

- Tree policy can be any exploration policy
- Backprop works just as before

Upper Confidence Bounds for Trees (UCT):

- Popular MCTS variant — easy to use and often effective
- Uses UCB action selection as tree policy, and $\alpha = 1/N(S, A)$

UCB recap: estimate upper bound on action value:

$$A \leftarrow \begin{cases} a, & \text{if } a \text{ never tried in } S \\ \arg\max_a Q(S, a) + c\sqrt{\log N(S)/N(S, a)} \end{cases}$$

- $N(S)$ is number of times state S has been visited
- $N(S, a)$ is number of times action a was selected in S

Simulation Step

Simulation step gives estimate of return at state, e.g.:

Random-DefaultPolicy(S):

```
G ← 0
while S is non-terminal do
    A ← random action (uniformly)
    (R, S′) ~ Model(S, A)
    G ← R + γG
    S ← S′
return G
```

Possible improvements:

- Average over multiple simulations
- Use domain-specific heuristic to
  – select better actions than random
  – evaluate state directly (e.g. in Chess we know that some states are better than others)

Imagine you are given an MDP for a chess game against a specific opponent

Offline planning:

- Use MDP to find best policy before the actual chess game takes place (offline)
- Use as much time as needed to find policy
- Policy is complete: gives optimal action for *all* possible states

Dyna-Q and dynamic programming are suitable for offline planning

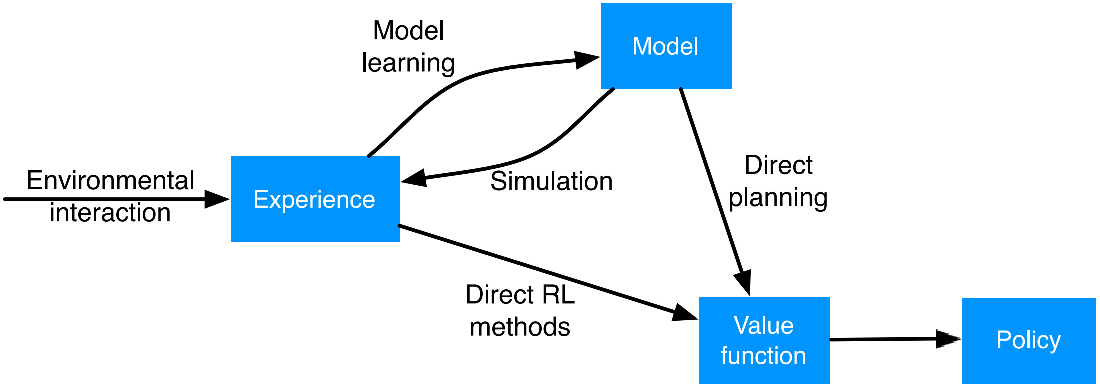
Imagine you are given an MDP for a chess game against a specific opponent

Online planning:

- Use MDP to find best policy during the actual chess game (online)
- Limited compute time budget at each state (e.g. seconds/minutes in chess)
- Policy usually incomplete: gives optimal action for *current* state

Rollout planning (including MCTS) is suitable for online planning

Paths to a Policy: Model-Based RL



Discussion: Model-based vs. model-free RL

- Models can provide additional information and thus increase efficiency and robustness.
- Models can be costly to obtain, to run, and to keep updated.
- Model-free approaches appear more interesting as they are more challenging, in particular when model learning is included.
- Both model-free and model-based approaches can have biases.

Reading

Required:

- RL book, Chapter 8 (8.1–8.3, 8.10–8.11)

Optional:

- Browne et al. (2012). A Survey of Monte Carlo Tree Search Methods. IEEE Transactions on Computational Intelligence and AI in Games, Vol. 4, No. 1
- UCT paper: L. Kocsis and C. Szepesvari (2006). Bandit based Monte-Carlo Planning. European Conference on Machine Learning
- T. Vodopivec, S. Samothrakis, B. Ster (2017). On Monte Carlo Tree Search and Reinforcement Learning. Journal of Artificial Intelligence Research, Vol. 60