Simulation, Analysis, and Validation of Computational Models

— Case study II: Models in computational finance —

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Overview

- Minority games
- Option pricing
- Market simulation
- Monte Carlo methods for option pricing

Minority games

- Once a week, a fixed population want to go have fun at the El Farol Bar (Santa Fe, New Mexico).
- If less than 60% of the population go to the bar, they'll all have more fun than if they stayed home.
- If more than 60% of the population go to the bar, they'll all have less fun than if they stayed home.
- Everyone must decide at the same time whether to go or not, with no knowledge of others' choices.

Original minority game

- Players $i \in \{1, \ldots, N\}$
- Actions $a_i \in \{-1, +1\} \ \forall i$
- Payoff:

$$u_i\left(a_i \,|\, \{a_j, j \neq i\}\right) = -a_i A \quad \text{where} \quad A = \sum_{i=1,\ldots,N} a_i$$

  depends on all actions but these are not known to player i
- Related problems
  - El Farol Bar problem
  - Kolkata Paise Restaurant Problem
  - Hospital bed problem
  - Stock market

Challet e.a. (2005) Minority Games: Interacting Agents in Financial Markets, OUP.

Minority games: Kolkata Paise Restaurant Problem

- N restaurants, n guests (usually $N = n$ is assumed)
- Each guest chooses once and independently of other guests
- A strategy is the probability to choose the k-th best restaurant based on the situation on the previous day (1-step memory) e.g. Boltzmann-type with two parameters α, T:

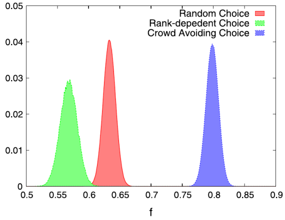
$$p_k(t) \propto k^\alpha \exp\left(-\frac{n_k(t-1)}{T}\right)$$

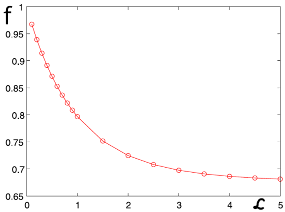
  for $n_k(t-1)$ customers last time in the k-th best place
- f: utilisation (average fraction of served guests)
- $\alpha = 0,\ T \to \infty$ (random choice): $f = 1 - \exp(-1) \approx 0.63$
- $\alpha = 1,\ T \to \infty$ (strict rank-dependent choice): $f \approx 0.57$
- $\alpha = 0,\ T \to 0$ (avoiding-past-crowd choice): $f \approx 0.46$

https://demonstrations.wolfram.com/KolkataPaiseRestaurantKPRProblem

Another type of strategies

- $p_k(t) \propto \frac{1}{n^{\alpha}_{k(t-1)}}$ (parameter α)
- Stochastic avoiding-past-crowd $p_k(t) \propto \frac{1}{n_{k(t-1)}}$: $f \approx 0.8$ for $\alpha = 1$
- Limit $\alpha \to 0$ is non-trivial: Converges to random choice for finite T, because for smaller α is takes longer to settle. For sufficiently long settling times, better and better utilisation is reached for $\alpha > 0$.
- $f = 0.676$ for $\alpha \to \infty$





Ghosh e.a. (2013) Kolkata Paise Restaurant Problem: An Introduction. In Econophysics, Springer.

Minority games: Effect of the memory depth

- If information from L previous steps can be taken into account by the agents, then (considering only up-down, i.e. two states) 2^{2^L} policies are possible.
- If the memory depth if flexible, then it will be tempting to increase memory, however, long-memory policies take a long time to get optimal, such that short policies can evolve quickly to take advantage of the inefficiency of the complex policies.
- If only a few policies (say D) can be stored, then a balance is reached below $D/L \approx 0.34$ [Patzold, 2014], while above this critical point the variance increases.

Financial markets

- Markets are different by country, industrial sectors, objects
- Agents have different wealth, time horizons, risk tolerance, expectations, and thus not all the same policy.
- Distribution of price returns ($\log p\,(t + \Delta t) - \log\,(t)$) is power-law for $\Delta t <$ day and Gaussian for $\Delta t >$ months
- Autocorrelation of returns is zero for $\Delta t >$ minutes
- Autocorrelation of absolute returns is positive for $\Delta t \approx$ day

Challet e.a. (2004) Minority games: Interacting agents in financial markets. OUP Oxford.

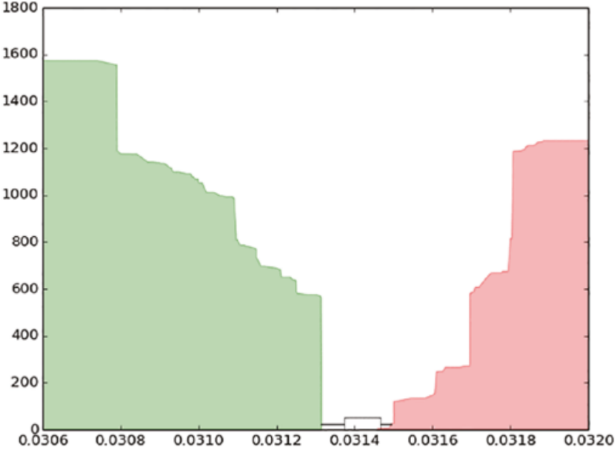
What to model?

- Investors tend to use "chartist" strategies that extrapolate short-term observations
- Mean-reverting "fundamentalist" investment strategies dominate on longer time horizons

Hommes & Wagener (2009) Complex evolutionary systems in behavioral finance. Elsevier.

Order book

Dynamic list of buy and sell orders updated by the match engine



Li & Nakano, Simulation with Python, Apress, Ch. 6

Stock trading process

Maintaining an order book

1. The investor sends an order to the match engine.
2. The match engine processes and tries to match the trade.
3. The match engine broadcasts messages to the original order submitters, if there is a successful trade.
4. The match engine broadcasts the latest prices to everyone in the market.

Li & Nakano, Ch. 6

Market efficiency

- Two markets, A and B, e.g. apples prices $p_A < p_B$.
- Buy apples at A and sell at B.
- Consequence: Demand at A increases as well as the supply at B.
- p_A will rise, and p_B will fall.
- Neglecting transportation costs (friction), prices will approach equilibrium where $p_A = p_B$.
- Is this always the case? If not, riskfree profit can be made called pure *arbitrage*.

Felix Patzold (2014) Instability and Information, PhD thesis

Market efficiency

- Market efficiency implies that stock markets are difficult to predict.
- Deep neural networks that employ step-wise linear regressions with exponential smoothing enable predictions purely based on lagged correlations between large numbers of time series.
- Tested on historical stock market data (S&P 500) from 2011 to 2016 without considering external information.
- Unlikely to work for today's data.
- Short-term predictions are still possible, if methods can be slower than real time.

Moews e.a. (2019) Lagged correlation-based deep learning for directional trend change prediction in financial time series. Exp. Syst. Appl, **120**,197-206.

Orders and options

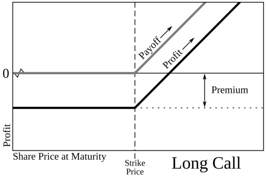
- Limit orders state the worst allowable price for the transaction
  - Sell orders with a lower price have priority over sell orders with a higher price.
  - Buy orders with a higher price have priority over buy orders with a lower price.
- Market orders are executed immediately at best available price
  - Market buy order will match sell orders from the lowest price to higher prices until a requested amount of shares is obtained (pushing up the market price)
  - Market sell order will match buy orders from the highest price to lower prices until a requested amount of shares is sold (lowering the market price)
- Options (European style), such as
  - The buyer of a *call option* has the right, but not the obligation, to buy an agreed quantity of an asset from the seller of the option at the expiration date for a certain price (the strike price)
  - The buyer of a *put* option has the right, but not the obligation, to sell an asset at a specified price (the strike price), on the expiration date.

Option prizing

- Example: Call option for Mince Pies at 1£, agreed in January with expiration time in November

- Cost for the transaction 20p
    - If the Mince Pie price in November is 2£, the buyer has gained 2£-1£-20p=80p (gain limited by risk)
    - If the Mince Pie price in November will be below 1£, the seller has gained 20p (gain limited by obligation to sell)

In theory, the buyer faces this situation:



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What is the value of the option in practice?

Black–Scholes model

- Mathematical model for the dynamics of a financial market
- Exponentiality is essential (it's about ratios): Price obeys geometric[1] Brownian Motion with constant drift μ, and constant volatility[2] σ (between placing and serving order: unknown at time of order, but assumed to be constant)

$$\frac{\mathrm{d}S}{S} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W.$$

- Black-Scholes equation (parabolic partial differential equation)

$$\frac{\partial V}{\partial t} + r\,S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2\,S^2\,\frac{\partial^2 V}{\partial S^2} = r\,V$$

  where V value, S price, r interest rate (fixed and known).
- Arbitrage not considered (no risk-free gains), no leaks and sources

Black & Scholes (1973), Merton (1973

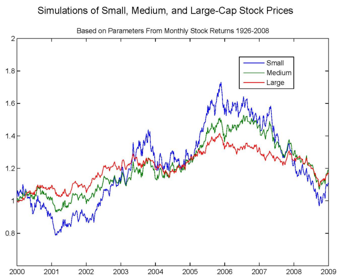
[1] i.e. Brownian motion of the logarithms.

[2] standard deviation of the logarithm of the value

Parabolic partial differential equations

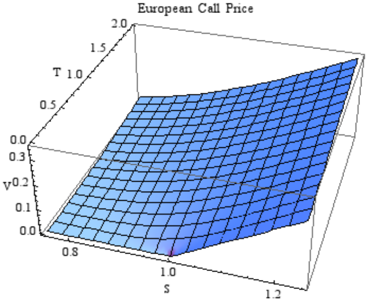
- First order in one variable (time), second order in the second variable (space).
- Formulation: Initial value problem or boundary value problem or mixed
- Numeric solution by (e.g.) the *Crank-Nicolson* method: Spatial discretisation by finite-differences and time discretisation by implicit trapezoid method.
- Many equations are of this type such as heat equation, Fokker-Planck equation, Navier-Stokes equation.

Stochastic process



Roberto Croce

Black-Scholes



Parsiad.azimzadeh, CC BY-SA 3.0, wikimedia.curid=22778960

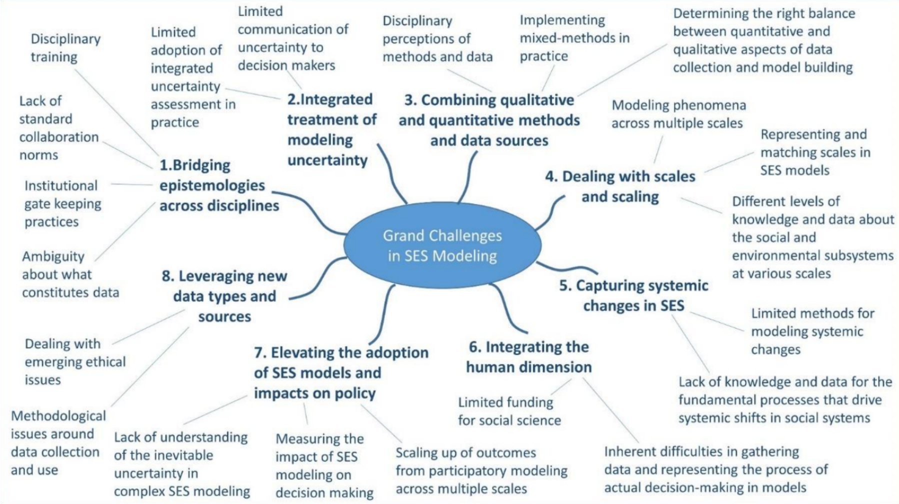
Black Scholes: Discussion

- Brownian motion (Gaussian) underestimates extreme events which can be endogenous (e.g. if all traders rely on the same incorrect model).
- Volatility can change, i.e. needs to be known, although the goal was to risk-free trade.
- Other assumptions need to be considered, but are less critical.
- Statement for short time intervals

Hommes & Wagener (2009) Complex evolutionary systems in behavioral finance. Elsevier.

- Many context factors need to be considered
  - Fundamental uncertainty (Frank Knight) E.g. type of distribution is not known, but can be known.
  - Radical uncertainty (Kay & King): event is unique or novel, so there knowledge does not (yet) exist.

Socio-environmental (socio-ecological) systems modelling



Elsawah e.a. (2020) 8 grand challenges in socio-environmental systems modeling. Socio-Envir. Syst. Model. 2,16226.

Next topics

- Simulations of social systems
- Large scale simulations
- Weather and climate