

# Automated Reasoning

## Solutions for Exercise Sheet 2: First Order Logic and Isabelle

### Exercise 1

See the Isabelle theory file.

Note: The statements from this exercise can be proved in several ways. Moreover, Isabelle has pre-proved lemmas that can make some of these proofs fairly trivial. You should attempt to prove the statements using only the basic Natural Deduction rules for classical FOL.

### Exercise 2

- $(\forall x.P x \rightarrow Q) \rightarrow (\exists x.P x \rightarrow Q)$

$$\frac{\frac{\frac{\overline{P a \rightarrow Q \vdash P a \rightarrow Q} \text{ assumption}}{\forall x.P x \rightarrow Q \vdash P a \rightarrow Q} \text{ allE}}{\forall x.P x \rightarrow Q \vdash \exists x.P x \rightarrow Q} \text{ exI}}{\vdash (\forall x.P x \rightarrow Q) \rightarrow (\exists x.P x \rightarrow Q)} \text{ impI}$$

- $\forall x.\neg P x$ , assuming that  $\neg\exists x.P x$

$$\frac{\frac{\frac{\overline{P x_0 \vdash P x_0} \text{ assumption}}{P x_0 \vdash \exists x.P x} \text{ exI}}{\neg\exists x.P x, P x_0 \vdash \perp} \text{ notE}}{\neg\exists x.P x \vdash \neg P x_0} \text{ notI}}{\neg\exists x.P x \vdash \forall x.\neg P x} \text{ allI}$$

3.  $\exists x.\neg P x$ , assuming that  $\neg\forall x.P x$  is true

$$\begin{array}{c}
 \frac{}{\neg P x_0 \vdash \neg P x_0} \text{assumption} \\
 \frac{\neg P x_0 \vdash \neg P x_0}{\neg P x_0 \vdash \exists x.\neg P x} \text{exI} \\
 \frac{\neg\exists x.\neg P x, \neg P x_0 \vdash \perp}{\neg\exists x.\neg P x \vdash P x_0} \text{notE} \\
 \frac{\neg\exists x.\neg P x \vdash P x_0}{\neg\exists x.\neg P x \vdash \forall x.P x} \text{ccontr} \\
 \frac{\neg\exists x.\neg P x \vdash \forall x.P x}{\neg\forall x.P x, \neg\exists x.\neg P x \vdash \perp} \text{allI} \\
 \frac{\neg\forall x.P x, \neg\exists x.\neg P x \vdash \perp}{\neg\forall x.P x \vdash \exists x.\neg P x} \text{notE} \\
 \frac{}{\neg\forall x.P x \vdash \exists x.\neg P x} \text{ccontr}
 \end{array}$$