Isabelle/HOL Exercises
Logic and Sets

Predicate Logic

We are again talking about proofs in the calculus of Natural Deduction. In addition to the rules given in the exercise "Propositional Logic", you may now also use

```
exI: P x ⟹ ∃x. P x
exE: ⟦∃x. P x; ⋀x. P x ⟹ Q⟧ ⟹ Q
allI: (⋀x. P x) ⟹ ∀x. P x
allE: ⟦∀x. P x; P x ⟹ R⟧ ⟹ R
```

Give a proof of the following propositions or an argument why the formula is not valid:

```
lemma "(∃x. ∀y. P x y) ⟶ (∀y. ∃x. P x y)"
lemma "(∀x. P x ⟶ Q) = ((∃x. P x) ⟶ Q)"
lemma "((∀ x. P x) ∧ (∀ x. Q x)) = (∀ x. (P x ∧ Q x))"
lemma "((∀ x. P x) ∨ (∀ x. Q x)) = (∀ x. (P x ∨ Q x))"
lemma "((∃ x. P x) ∨ (∃ x. Q x)) = (∃ x. (P x ∨ Q x))"
lemma "(∀x. ∃y. P x y) ⟶ (∃y. ∀x. P x y)"
lemma "(¬ (∀ x. P x)) = (∃ x. ¬ P x)"
```