Isabelle/HOL Exercises
Logic and Sets

Propositional Logic

In this exercise, we will prove some lemmas of propositional logic with the aid of a calculus of natural deduction.

For the proofs, you may only use

- the following lemmas:
  notI: $(A \Longrightarrow False) \Longrightarrow \neg A$,
  notE: $[\![\neg A; A]\!] \Longrightarrow B$,
  conjI: $[\![A; B]\!] \Longrightarrow A \wedge B$,
  conjE: $[\![A \wedge B; [\![A; B]\!] \Longrightarrow C]\!] \Longrightarrow C$,
  disjI1: $A \Longrightarrow A \vee B$,
  disjI2: $A \Longrightarrow B \vee A$,
  disjE: $[\![A \vee B; A \Longrightarrow C; B \Longrightarrow C]\!] \Longrightarrow C$,
  impI: $(A \Longrightarrow B) \Longrightarrow A \longrightarrow B$,
  impE: $[\![A \longrightarrow B; A; B \Longrightarrow C]\!] \Longrightarrow C$,
  mp: $[\![A \longrightarrow B; A]\!] \Longrightarrow B$
  iffI: $[\![A \Longrightarrow B; B \Longrightarrow A]\!] \Longrightarrow A = B$,
  iffE: $[\![A = B; [\![A \longrightarrow B; B \longrightarrow A]\!] \Longrightarrow C]\!] \Longrightarrow C$
  classical: $(\neg A \Longrightarrow A) \Longrightarrow A$
- the proof methods *rule*, *erule* and *assumption*.

Prove:

lemma *I:* "$A \longrightarrow A$"
lemma "$A \wedge B \longrightarrow B \wedge A$"
lemma "$(A \wedge B) \longrightarrow (A \vee B)$"
lemma "$((A \vee B) \vee C) \longrightarrow A \vee (B \vee C)$"
lemma *K:* "$A \longrightarrow B \longrightarrow A$"
lemma "$(A \vee A) = (A \wedge A)$"
lemma *S:* "$(A \longrightarrow B \longrightarrow C) \longrightarrow (A \longrightarrow B) \longrightarrow A \longrightarrow C$"
lemma "$(A \longrightarrow B) \longrightarrow (B \longrightarrow C) \longrightarrow A \longrightarrow C$"
lemma "$\neg \neg A \longrightarrow A$"
lemma "$A \longrightarrow \neg \neg A$"
lemma "$(\neg A \longrightarrow B) \longrightarrow (\neg B \longrightarrow A)$"

lemma *"((A ⟶ B) ⟶ A) ⟶ A"*
lemma *"A ∨ ¬ A"*
lemma *"(¬ (A ∧ B)) = (¬ A ∨ ¬ B)"*