

Automated Reasoning

Exercise Sheet 4: Structured Proofs in Isabelle/HOL

Exercise 1

Provide structured (declarative) proofs the following statements in Isabelle/Isar:

- $(P \longrightarrow (Q \longrightarrow R)) \longrightarrow ((P \longrightarrow Q) \longrightarrow (P \longrightarrow R))$
- $(\forall x. P\ x \longrightarrow Q) \longrightarrow (\exists x. P\ x \longrightarrow Q)$
- $\forall x. \neg P\ x$, assuming that $\neg \exists x. P\ x$ is true
- $\exists x. \neg P\ x$, assuming that $\neg \forall x. P\ x$ is true
- $(R \longrightarrow P) \longrightarrow (((\neg R \vee P) \longrightarrow (Q \longrightarrow S)) \longrightarrow (Q \longrightarrow S))$

Exercise 2

Provide a structured proof of the following theorem (from the additional Isabelle exercises) using Isabelle/Isar. Use case distinctions and/or proof by contradiction.

$$(\forall x. \neg rich\ x \longrightarrow rich\ (father\ x)) \longrightarrow \exists x. rich\ (father\ (father\ x)) \wedge rich\ x$$

Exercise 3

This question revisits the Euclidean geometry problem from Tutorial 3. Give structured proofs of the following statements using Isar. Your proofs should not be 1-line ones obtained using Sledgehammer. You should lay out the reasoning explicitly using proof blocks.

- Not all points lie on the same line.
- There exist at least two lines through each point.
- Two lines cannot intersect in more than one point.